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SUBGRADE ELASTIC MODULI DETERMINED FROM VIBRATORY TESTING OF PAVEMENTS

Richard A. Weiss

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Soils and Pavements Laboratory
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OCTOBER 1977
FINAL REPORT



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12. Abstract <p>A layered elastic theory approach to the calculation of the allowable load-carrying capacity and required overlay thickness of a pavement requires the values of the static elastic moduli of the pavement layers and the subgrade. A method is developed for determining the subgrade Young's modulus by vibratory nondestructive tests performed on the pavement surface. The response of pavements and subgrades to dynamic loadings is nonlinear, so that a nonlinear dynamic model of pavement response was developed by which the subgrade Young's modulus can be obtained from the measured dynamic pavement response data. The value of the subgrade Young's modulus predicted by vibratory nondestructive field tests was checked by laboratory dynamic resilient modulus tests. This was done by developing a nonlinear theory of the dynamic resilient modulus test by which the Young's modulus of the subgrade soil sample can be extracted from the resilient modulus test data.</p>		13. Type of Report and Period Covered Final report, Oct 74-Sep 76
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PREFACE

This study was conducted during the period October 1974-September 1976 by personnel of the U. S. Army Engineer Waterways Experiment Station (WES). It was sponsored by the Federal Aviation Administration through part of Inter-Agency Agreement DOT FA73WAI-377, "New Pavement Design Methodology."

The study was done under the general supervision of Messrs. James P. Sale and Richard G. Ahlvin, Chief and Assistant Chief, respectively, of the Soils and Pavements Laboratory, Mr. Ronald L. Hutchinson, Pavement Program Manager, and Mr. Harry H. Ulery, Jr., Chief of the Pavement Design Division, and under the direct supervision of Mr. Alfred H. Joseph, Chief of the Pavement Investigations Division, and Mr. Jim W. Hall, Jr., Chief of the Evaluation Branch. The programming for this study was done in part by Mr. Arden P. Park, Soils Testing Branch. Significant contributions were made by Mr. James L. Green, Evaluation Branch. This report was written by Dr. Richard A. Weiss.

COL G. H. Hilt, CE, and COL J. L. Cannon, CE, were Directors of WES during the conduct of this study and the preparation of this report. The Technical Director was Mr. F. R. Brown.

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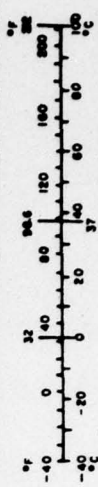
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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures			
Symbol	When You Know	Multiply by	To Find
LENGTH			
in	inches	2.5	centimeters
ft	feet	30	centimeters
yd	yards	0.9	meters
mi	miles	1.6	kilometers
AREA			
sq in	square inches	6.5	square centimeters
sq ft	square feet	0.09	square meters
sq yd	square yards	0.8	square meters
sq mi	square miles	2.6	square kilometers
ac	acres	0.4	hectares
MASS (weight)			
oz	ounces	28	grams
lb	pounds	0.45	kilograms
sh	short tons (2000 lb)	0.9	tonnes
VOLUME			
cup	teaspoons	5	milliliters
fl oz	tablespoons	15	milliliters
c	fluid ounces	30	milliliters
pt	cups	0.24	liters
qt	pints	0.47	liters
gal	quarts	0.95	liters
cu ft	gallons	3.8	liters
cu yd	cubic feet	0.03	cubic meters
	cubic yards	0.76	cubic meters
TEMPERATURE (exact)			
F	Fahrenheit temperature	5/9 after subtracting 32	Celsius temperature
C	Celsius temperature	9/5 then add 32	Fahrenheit temperature

* 1 in. = 2.54 (exactly). For other exact conversions and more detailed tables, see NBC Misc. Publ. 286, Units of Weights and Measures, Price \$2.25, SD Catalog No. C13.10.286.

Approximate Conversions to Metric Measures			
Symbol	When You Know	Multiply by	To Find
LENGTH			
mm	millimeters	0.04	inches
cm	centimeters	0.4	inches
m	meters	3.3	feet
km	kilometers	1.1	miles
		0.6	miles
AREA			
sq cm	square centimeters	0.16	square inches
sq m	square meters	1.2	square yards
sq km	square kilometers	0.4	square miles
ha	hectares (10,000 m ²)	2.5	acres
MASS (weight)			
g	grams	0.005	ounces
kg	kilograms	2.2	pounds
t	tonnes (1000 kg)	1.1	short tons
VOLUME			
ml	milliliters	0.03	fluid ounces
l	liters	2.1	pints
		1.06	quarts
		0.26	gallons
m ³	cubic meters	36	cubic feet
		1.3	cubic yards
TEMPERATURE (exact)			
F	Fahrenheit temperature	5/9 then add 32	Celsius temperature
C	Celsius temperature	9/5 then add 32	Fahrenheit temperature



1. INTRODUCTION

1.1 BACKGROUND

Because pavement performance is dependent on strength, which is often expressed in terms of a limiting vertical strain in the subgrade for asphaltic concrete (AC) pavements and a limiting tensile stress in the rigid pavement layer for portland cement concrete (PCC) pavements, it is necessary to employ a layered elastic approach to the problems of calculating the load-carrying capacity of a pavement and the overlay thickness required for a pavement. The values of the elastic moduli of the subgrade and each pavement layer are required for this approach, and fast and reliable methods for determining the in situ elastic moduli of a pavement-subgrade system are also required. It is generally thought that the value of the in situ subgrade Young's modulus is the significant unknown parameter to be determined by the method of vibratory non-destructive testing of pavements.

The U. S. Army Engineer Waterways Experiment Station (WES) has for many years used the method of vibratory nondestructive testing of airfield pavements.¹⁻⁵ This method is relatively quick, reproducible, and inexpensive. Nondestructive vibratory testing of pavements may eventually lead to a very accurate prediction of load-carrying capacity and pavement life. The Young's modulus of the subgrade is closely related to load-bearing capacity and pavement life and is an important factor in the design of pavements and overlays. Therefore, it is important to have a method of determining the Young's modulus of the subgrade by performing vibratory nondestructive tests at the pavement surface.

The allowable static load (load-carrying capacity) for a pavement is related to a limiting value of the vertical strain in the subgrade of an asphaltic-concrete pavement and to a limiting tensile stress in the wearing surface of a rigid pavement. Layered elastic theory relates the limiting strain and stress values to the allowable load at the pavement surface, and computer programs are available to do this. These static elastic computer programs require the Poisson's ratio and Young's modulus for each pavement layer and subgrade. For a specified Poisson's

ratio, it is Young's modulus of a material that describes its response to an applied static load.

It is important to be able to predict the Young's modulus of the subgrade directly from the dynamic response data produced by the vibratory nondestructive tests conducted at the surface of a pavement, because it is the static elastic Young's modulus that is entered in the layered elastic computer programs for determining the allowable load for a pavement. Because soils behave in a nonlinear manner under dynamic and static loadings, the static and dynamic loads produced by a vibrator will affect the elastic properties of the soil in the subgrade. The subgrade Young's modulus that is entered in the allowable load calculation must be independent of the static and dynamic loads generated by the vibrator that is used for the nondestructive testing of a pavement.

The subgrade Young's modulus that is entered in the layered elastic computer program for pavement evaluation must depend only on the static confining pressure produced in the subgrade by the aircraft load applied to the pavement surface and by the natural overburden pressure in the subgrade. Extraction of the subgrade Young's modulus from dynamic vibratory response data taken in the field requires a nonlinear dynamic theory of pavement response which can isolate the effects of the static and dynamic loads generated by the vibrator.

An independent method of checking the values of the subgrade Young's modulus that are predicted by vibratory nondestructive field tests would give pavement engineers the confidence to use these modulus values for pavement evaluation. The subgrade Young's modulus that is determined from the vibratory nondestructive field tests must agree with the Young's modulus value that is obtained from laboratory tests done on an undisturbed soil sample taken from the subgrade of a pavement. The laboratory test considered is the resilient modulus test, which is a dynamic test done on a soil sample for a specific static confining pressure. The resilient modulus is a measure of the response of a soil to a dynamic load; i.e., the resilient modulus is a dynamic modulus. The resilient modulus is a nonlinear function of the applied dynamic deviator stress, and therefore a nonlinear dynamic theory of the resilient

modulus test is required to extract the value of the static elastic Young's modulus from the dynamic test data obtained in the laboratory.

The research presented in this report is part of a more extensive research program in which pavement performance will be predicted by the layered elastic model for a pavement and subgrade whose elastic moduli are obtained by vibratory nondestructive field tests. Wave propagation methods exist that determine the Young's modulus of each pavement layer as well as the Young's modulus of the soil in the subgrade of a pavement. In this report, it is assumed that the elastic moduli of the pavement layers are known and only the subgrade Young's modulus must be determined by vibratory nondestructive testing methods. Eventually the elastic moduli of all the pavement layers and the subgrade may be determined by vibratory nondestructive testing techniques.

Experimental and theoretical investigations were performed to find methods for determining the subgrade Young's modulus by vibratory nondestructive field test methods and to correlate the field test results with laboratory tests done on undisturbed subgrade soil samples.

The instrument used for the vibratory nondestructive testing of pavements was a mechanical vibrator whose force payload to the pavement surface is generated either by a hydraulic system or a mechanism of counterrotating weights. The WES 16-kip* vibrator applies a static load of 16 kips to the pavement surface and a dynamic load up to 15 kips at frequencies ranging from 5 to 100 Hz. Both static and dynamic loads are applied to the pavement surface through a circular 18-in.-diam baseplate.

Four types of nondestructive tests are generally performed on pavements, and these consist of the following measurements:

- a. Dynamic load-deflection curves giving the dynamic amplitude as a function of the dynamic load.
- b. Frequency response spectrum giving the dynamic amplitude as a function of frequency for a fixed dynamic load.
- c. Deflection basin measurements.
- d. Rayleigh wave dispersion curves giving phase velocity versus wavelength.

* A table for converting units of measurement is presented on page 5.

Only the impedance methods--the dynamic load-deflection curves (a above) and the frequency response spectrum measurements (b above)--will be considered in detail in this report, and two methods of determining the subgrade Young's modulus based on these measurements will be examined.

1.2 OBJECTIVES

The basic objectives of this study are:

- a. Development of a procedure for determining the Young's modulus of a subgrade of a pavement using vibratory nondestructive test data taken at the surface of the pavement.
- b. Development of a method of laboratory confirmation of the subgrade Young's moduli values measured in the field.

The study of the determination of the subgrade Young's modulus and its connection with the resilient modulus measured in the laboratory includes the following specific objectives:

- a. Development of a linear elastic dynamic model to describe the frequency response measurements and to determine the subgrade Young's modulus from these measurements.
- b. Development of a nonlinear elastic dynamic model of pavement response to describe the measured nonlinear dynamic load-deflection curves and to determine the subgrade Young's modulus from this type of field measurement.
- c. Development of a dynamic model which will analytically describe the nonlinear dependence of the laboratory resilient modulus on the static confining pressure and the dynamic deviator stress and which gives the procedure for extracting the Young's modulus from the resilient modulus.
- d. Determination of a procedure for comparing and correlating laboratory and field test values of the Young's modulus, and the development of the capability of extrapolating the laboratory derived Young's modulus to values of the static confining pressure that are expected to occur in the subgrade for an actual aircraft loading.

Development of a linear elastic dynamic model to describe the frequency response spectrum measured at a pavement surface should be regarded as the first step toward development of a nonlinear elastic dynamic model of the frequency response curves. The nonlinear model of the frequency response spectrum can be obtained from development of the nonlinear model of the dynamic load-deflection curves; however, this

analysis is rather complicated and is still under development. A non-linear dynamic theory is required for the frequency response spectrum measurements and the load-deflection curve data in order to remove the extraneous effects of the static and dynamic loads generated by the WES 16-kip vibrator on the value of the predicted subgrade Young's modulus. A computer program ECNST, based on the linear elastic theory, was developed to determine the subgrade Young's modulus from the measured frequency response curves.

The resilient modulus is a measure of the dynamic response of a material and cannot be used directly in the static layered elastic computer programs that calculate the static load-carrying capacity of a pavement. The static elastic Young's modulus must be extracted from the laboratory resilient modulus test data. It is this laboratory derived Young's modulus evaluated at a confining pressure equal to the overburden pressure in the subgrade that must be compared with the subgrade Young's modulus that is derived from vibratory nondestructive field test data. The value of the subgrade Young's modulus that enters the layered elastic computer programs for calculating the allowable load-carrying capacity of a pavement is the laboratory derived Young's modulus extrapolated to a value of the confining pressure that is expected to occur in the subgrade due to the actual static weight of an aircraft.

1.3 SCOPE

To achieve the objectives listed above, theoretical and experimental studies were performed.

1.3.1 THEORETICAL STUDIES

The theoretical studies included:

- a. Development of a technique for determining the subgrade Young's modulus by applying the linear elastic pavement response model to the frequency response spectrum measured at the pavement surface. This includes the determination of the inertial, damping, and elastic parameters directly from the measured frequency response curves.
- b. Development of a method for determining the subgrade Young's modulus from measured nonlinear dynamic load-deflection curves by using the nonlinear elastic pavement response model

to describe the measured dynamic load-deflection curves. This requires the determination of the inertial, damping, and elastic parameters of the model from the measured load-deflection curves.

- c. Determination of the parameters which are entered in the nonlinear dynamic model of the resilient modulus laboratory tests and which describe the dependence of the Young's modulus (derived from laboratory tests) on the static confining pressure applied to the soil sample.

The basic purpose of the theoretical studies described in this report is the development of a nonlinear elastic model of pavement response to dynamic and static loads. This model will give a theoretical expression for the dynamic load-deflection curves and the frequency response spectrum which will depend on: the applied dynamic and static loads, the Young's modulus, Poisson's ratio, and thickness of each pavement layer including the subgrade and on a set of pavement parameters that describe the inertial, damping, and nonlinear elastic behavior of a pavement. If the values of the elastic moduli of the upper pavement layers are known and if the values of the nonlinear elastic parameters are known for a pavement site, then the value of the subgrade Young's modulus at the pavement site can be found by requiring that the theoretical pavement response be equal to the measured pavement response.

When this procedure is applied to the frequency response spectrum using the linear elastic dynamic model, it is not possible to separate the effects of the static and dynamic loads from the predicted value of the subgrade Young's modulus. The linear elastic dynamic theory applied to the frequency response spectrum produces values of the effective mass, damping coefficient, and elastic spring constant, but the value of the elastic spring constant is conditioned by the static and dynamic loads generated by the vibrator and so the value of the predicted Young's modulus will also be conditioned by these extraneous loads. The value of the subgrade Young's modulus of physical interest depends only on the natural overburden pressure in the subgrade, and a nonlinear theory of the frequency response spectrum is required to remove the extraneous effects of the static and dynamic loads generated by the vibrator. The nonlinear theory of the frequency response spectrum is still under development.

The nonlinear dynamic theory presented in this report was developed to describe the nonlinear dynamic load-deflection curves. In this model, in addition to the Young's moduli, Poisson's ratios, and layer thicknesses, there also occur parameters that describe the inertial, damping, and nonlinear elastic behavior of the pavement materials. These parameters must be determined by fitting the theoretical model to pavements of known structure. Once these parameters are determined, the theoretical model will depend only on the elastic moduli of the pavement layers and subgrade and on the static and dynamic loads generated by the vibrator. In this way, the static and dynamic loads generated by the vibrator are separated from the elastic moduli of the pavement and subgrade, and the elastic moduli will depend only on the natural overburden pressure. The Young's modulus of the subgrade is then obtained by matching the theoretical dynamic load-deflection curve with the measured load-deflection curve. A computer program SUBE was developed to calculate the subgrade Young's modulus from the measured dynamic load-deflection curves.

It is important to be able to relate the Young's modulus value derived from laboratory resilient modulus tests on subgrade soil to the subgrade Young's modulus value that is predicted by the dynamic model applied to vibratory nondestructive field test data. The resilient modulus measures the response of a material to a dynamic load, and the theoretical model that describes the resilient modulus in terms of dynamic deviator stress and the static confining pressure must contain material parameters that describe this dependence. These material parameters must be known if the effects of the dynamic load are to be removed and the static-pressure-dependent Young's modulus is to be extracted from the dynamic resilient modulus test data. The dependence of the Young's modulus on the static confining pressure must be known because this modulus must be evaluated at the natural overburden pressure in the subgrade if a comparison is to be made with the subgrade Young's modulus that is obtained by vibratory nondestructive field tests.

For use in the layered elastic theory computer programs that calculate the allowable load-carrying capacity of a pavement, the

laboratory derived Young's modulus must be extrapolated to a value of the static confining pressure that is equal to the confining pressure produced in the subgrade by the static weight of the aircraft plus the overburden pressure in the subgrade.

1.3.2 EXPERIMENTAL STUDIES

The experimental field studies were performed on pavements and subgrades, and the experimental laboratory studies were resilient modulus tests on undisturbed subgrade soil samples. The experimental studies included:

- a. Measurement of dynamic load-deflection curves using a vibrator developed at WES which can generate dynamic loads up to 15 kips at a frequency of 15 Hz and with a constant 16-kip static load (WES 16-kip vibrator).
- b. Measurement of dynamic frequency response curves giving dynamic amplitude versus frequency for a constant dynamic loading.
- c. Laboratory measurement of resilient moduli of subgrade soils for a series of static confining pressures and dynamic deviator stresses.

The theoretical and experimental work done in this report will have applications for the nondestructive testing of roads and airport pavements. The dynamic load-deflection curves measured in the field can be used to determine the subsurface structure, and if the elastic moduli of the pavement layers are assumed to be known, the subgrade Young's modulus can be determined.

2. DYNAMIC FREQUENCY RESPONSE SPECTRUM METHOD FOR DETERMINING THE SUBGRADE MODULUS

2.1 GENERAL CONSIDERATIONS

The problem of calculating the allowable load of a pavement or the required overlay thickness of a pavement can be treated by using a layered elastic interpretation of pavement response. This method requires that the elastic moduli of each pavement layer be known. The allowable load or pavement overlay thickness is calculated for flexible pavements by specifying a limiting strain in the soil of the subgrade, and for rigid pavements by specifying a limiting stress at the bottom of the rigid pavement layer. A layered elastic computer program is utilized to relate the load applied at the pavement surface to the stress and strain in the pavement layers and subgrade. The required input parameters for this computer program are the elastic moduli of the subgrade and the elastic moduli and thickness of each pavement layer. With a knowledge of these input parameters, the limiting stress and strain conditions can be transformed into allowable loads and pavement overlay thicknesses.

A quick method of determining the elastic moduli of the pavement layers and subgrade is desirable. Various techniques have been used for determining all of the elastic moduli by using vibratory nondestructive testing methods.^{3,4} For instance, the wave propagation method utilizing Rayleigh waves has been used, but with limited success. The Young's modulus value of the subgrade is generally known with less precision than the modulus values of the pavement layers. The elastic moduli of the wearing surface, base, and subbase of a pavement can be obtained from a knowledge of the type of material in these layers and from laboratory tests such as the resilient modulus test. The Young's modulus of PCC pavement is known with reasonable precision to vary from 4.0 to 6.0×10^6 psi; the Young's modulus of AC pavements is known to be temperature-dependent from laboratory tests; and the elastic moduli of base and subbase materials can be estimated from laboratory resilient modulus tests.

A reasonable procedure for the vibratory nondestructive evaluation of pavements that utilizes the layered elastic approach would be to consider the elastic moduli of the pavement layers to be known, and treat the subgrade Young's modulus as the unknown quantity to be determined from the vibratory nondestructive test data. The frequency response spectrum method of determining the subgrade Young's modulus is outlined in this section. This section develops a simple spring model interpretation of the measured frequency response data and determines a spring constant for the entire pavement and subgrade system. The spring constant is then related to a value of the Young's modulus of the subgrade by using the Chevron layered elastic computer program in which the elastic moduli of all of the pavement layers have been previously selected. This method of predicting the subgrade Young's modulus gives modulus values which are considerably higher than those predicted by the Shell relationship, which relates the linear elastic Young's modulus to the measured CBR of a subgrade.

The mechanical vibrator that is used for the nondestructive testing of pavements and subgrades operates at a known frequency and produces a sinusoidal dynamic force and dynamic deflection of the pavement surface directly beneath the vibrator baseplate. The frequency response spectrum gives the dynamic deflection of the pavement surface beneath the vibrator baseplate as a function of frequency for a fixed value of the dynamic load. A typical measured frequency response spectrum appears in Figure 1. This section investigates a method for determining the subgrade Young's modulus from the frequency response spectrum measured at a pavement surface. This method includes the development of a linear spring-mass-dashpot model to describe the measured frequency response spectrum. The elements of this model (spring constant, damping constant, and effective mass) are determined directly from the measured maximum amplitude and the frequency at maximum amplitude. The value of the subgrade Young's modulus is then calculated from the static spring constant by using a standard linear layered elastic half-space computer program.

It is not immediately evident whether the pavement-subgrade system

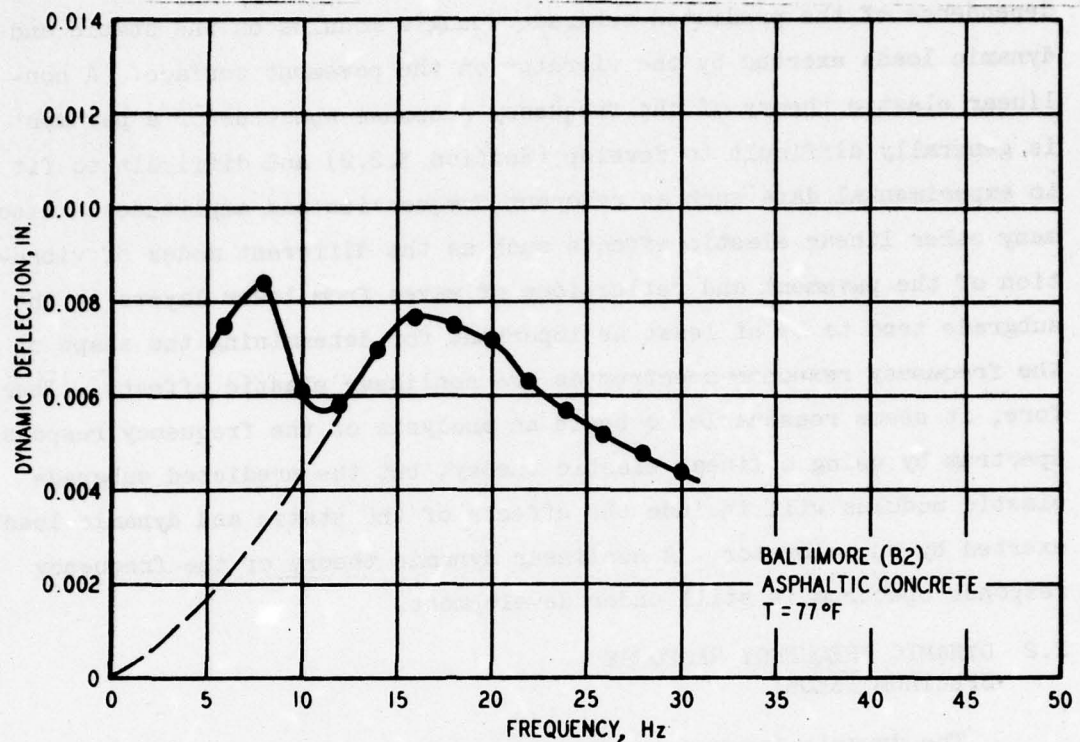


Figure 1. Typical frequency response curve

is linear elastic or nonlinear elastic by examining the frequency response spectrum that is determined for a fixed value of the dynamic load. However, it is known that pavement and subgrade materials have nonlinear elastic load-deflection curves for static and dynamic loadings. These nonlinear elastic properties require the elastic moduli of the pavement and subgrade to be dependent on the magnitude of the static and dynamic stress produced by the vibrator in the pavement and subgrade. The value of the subgrade Young's modulus that is theoretically predicted from a frequency response spectrum for fixed dynamic and static loads is dependent on the magnitude of these loads. A direct comparison of the subgrade Young's modulus predicted from the frequency response spectrum with the subgrade Young's modulus predicted by wave propagation methods (Shell formula) need not produce agreement.

A nonlinear elastic theory of the frequency response spectrum of pavements subjected to dynamic loads is required to characterize the

dependence of the predicted subgrade Young's modulus on the static and dynamic loads exerted by the vibrator on the pavement surface. A nonlinear elastic theory of the frequency response spectrum of a pavement is generally difficult to develop (Section 3.2.2) and difficult to fit to experimental data such as resonant frequencies and amplitudes. Also, many other linear elastic effects such as the different modes of vibration of the pavement and reflections of waves from lower layers in the subgrade tend to be at least as important for determining the shape of the frequency response spectrum as are nonlinear elastic effects. Therefore, it seems reasonable to begin an analysis of the frequency response spectrum by using a linear elastic theory, but the predicted subgrade elastic modulus will include the effects of the static and dynamic loads exerted by the vibrator. A nonlinear dynamic theory of the frequency response spectrum is still under development.

2.2 DYNAMIC FREQUENCY RESPONSE SPECTRUM THEORY

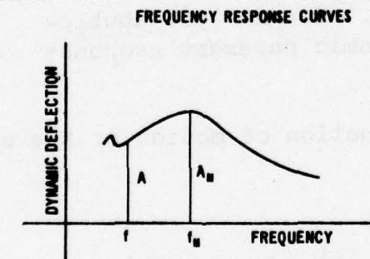
The dynamic frequency response spectrum measured at the pavement surface is often quite complex and difficult to interpret. Many factors probably contribute to produce its characteristic shape. The measured frequency response spectrum for a flexible or rigid pavement has more than one deflection peak. The physical origin of these peaks is difficult to determine with certainty, but they may be due to the different possible modes of vibration of a pavement. In order to extract some information about pavement and subgrade structure from the measured dynamic frequency response spectrum, it is necessary to use a simple dynamic pavement response model to fit the measured frequency response spectrum with the theoretically predicted frequency response spectrum. This fit will yield the parameters of the dynamic model from which the pavement and subgrade structure can be determined.

The simplest mechanical model that has a peak in its frequency response spectrum is the mass-spring-dashpot model (also called the Kelvin model). The frequency response spectrum of the Kelvin model exhibits only one deflection peak. Therefore, this model cannot describe

the complicated measured frequency response spectra of pavements. Nevertheless, some useful information can be obtained by fitting the Kelvin model individually to several of the observed deflection peaks of a frequency response spectrum measured at the surface of a pavement.

The first and second deflection peaks are generally the most pronounced. The first deflection peak generally occurs at about 8 Hz for all types of pavements and subgrades. This first peak is often eliminated by the electronic filtering that is used during the measurement of the frequency response spectrum, but the second peak is found to be unaffected by the electronic filtering equipment. How much of the amplitude of the first peak is due to electronic manipulation is still not resolved.

Each deflection peak is associated with a maximum amplitude and a frequency at maximum amplitude as shown in Figure 2 for the case of the second peak. The maximum amplitude and frequency were used to calculate the elements of the spring model: effective mass, effective spring constant, and effective damping constant. The elements of the Kelvin model can be simply related to the deflection peak. The Kelvin model was applied individually to the first and second deflection peaks, and it was found that the model parameters obtained from each peak were roughly the



MEASURED QUANTITIES: f_n = PEAK FREQUENCY
 A_n = PEAK DYNAMIC DEFLECTION
 $J(f) = A_n / A$ - RATIO OF PEAK DEFLECTION
 TO DEFLECTION AT
 ARBITRARY FREQUENCY

Figure 2. Measured quantities obtained from frequency response curves

same. This indicated that the entire pavement structure is responsible for each deflection peak and that, for instance, the first peak cannot be interpreted as being due solely to the subgrade.

2.3 LINEAR SPRING MODEL

The simplest mechanical model that can be used to describe the dynamic response of a pavement that is subjected to a sinusoidal force applied to the surface of the pavement is the linear Kelvin model that is shown in Figure 3. Linear spring models can be single-mass or

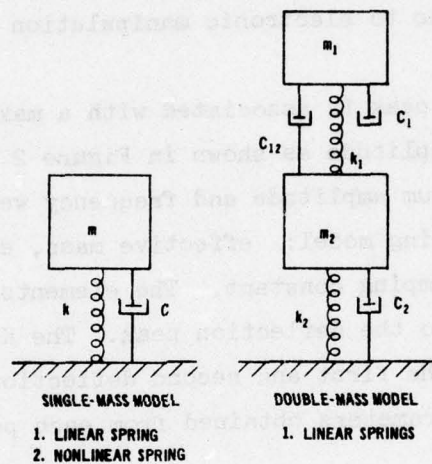


Figure 3. Single- and double-mass dynamic pavement response models

multiple-mass models. The equation of motion of the single-mass model is given by⁶⁻⁹

$$m\ddot{A} + C\dot{A} + kA = F_D(t) \quad (2.1)$$

where

- m = lumped effective mass
- \ddot{A} = acceleration of pavement surface
- C = damping constant
- \dot{A} = velocity of pavement surface
- k = spring constant

A = dynamic amplitude of pavement surface

$F_D(t)$ = dynamic force applied to the pavement surface

t = time

For a sinusoidal loading, the dynamic force is given by

$$F_D(t) = F_D(\omega)e^{i\omega t} \quad (2.2)$$

where

$F_D(\omega)$ = magnitude of the sinusoidal dynamic force applied to the pavement surface

$e^{i\omega t}$ = complex number notation for a sinusoidal time dependence

where $i = \sqrt{-1}$ and ω = angular frequency

Combining Equations 2.1 and 2.2 gives the dynamic deflection as

$$A = \frac{F_D(\omega)e^{i(\omega t - \Lambda)}}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}} \quad (2.3)$$

where Λ is phase angle between the dynamic load applied to the pavement surface and the dynamic deflection of the pavement surface, and is given by $\tan \Lambda = C\omega/(k - m\omega^2)$.

The frequency at which the deflection has a maximum is called the frequency at maximum amplitude or peak frequency. The standard method of determining the elements k , m , and C involves the measurement of the peak frequency, the peak amplitude, and the resonance frequency. The resonance frequency occurs when the phase angle Λ equals $\pi/2$. These three measured quantities are sufficient to determine the three unknown elements k , m , and C . Because no phase angles between the displacement of the pavement surface and the applied dynamic load have been measured, an alternative method, not requiring the measured phase angle, is developed for determining k , m , and C .

Within the framework of the single-mass spring model the dynamic amplitude of the pavement surface response to a sinusoidal dynamic load can be written as ⁶⁻⁹

$$A = \frac{F_D}{S} \quad (2.4)$$

$$S = \sqrt{(k - m\omega^2)^2 + c^2\omega^2} \quad (2.5)$$

where S is dynamic stiffness of the pavement surface. The peak frequency and amplitude can be obtained from Equations 2.4 and 2.5, for the case of constant F_D to be⁶

$$f_M = f_R \sqrt{1 - 2D^2} \quad (2.6)$$

$$A_M = \frac{F_D}{2kD \sqrt{1 - D^2}} \quad (2.7)$$

$$D = \frac{c}{2\sqrt{km}} \quad (2.8)$$

where

f_M = peak frequency

f_R = resonance frequency = $\sqrt{k/m}/2\pi$

A_M = peak amplitude

D = damping ratio

The three elements of the linear spring model that are to be obtained are k , m , and C . These three quantities can be obtained from measurements of f_M , f_R , and A_M . The resonance frequency f_R was not measured. In order to determine these three parameters, another piece of information, in addition to f_M and A_M , is necessary. This is given by

$$J(\omega) = \frac{A_M}{A} \quad (2.9)$$

where $J(\omega)$ is ratio of the peak amplitude to the amplitude at some nearby frequency. The theoretical value of this ratio is given by

$$J(k,m,C,\omega) = \frac{\sqrt{(k - m\omega^2)^2 + C^2\omega^2}}{\sqrt{(k - m\omega_M^2)^2 + C^2\omega_M^2}} \quad (2.10)$$

The three measured quantities which are extracted from the frequency response curve are f_M , A_M , and $J(\omega)$.

The spring model elements k , m , and C must now be obtained in terms of f_M , A_M , and $J(\omega)$. Equations 2.6, 2.7, and 2.8 can be inverted to determine k and D in the following manner

$$k = 4\pi^2 m f_M^2 \sqrt{1 + \left(\frac{F_D}{4\pi^2 m f_M^2 A_M} \right)^2} \quad (2.11)$$

$$D^2 = \frac{1}{2} - \frac{1}{2} \left[1 + \left(\frac{F_D}{4\pi^2 m f_M^2 A_M} \right)^2 \right]^{-1/2} \quad (2.12)$$

The k and D terms have now been expressed in terms of the effective mass. Using Equations 2.11 and 2.12, it is now possible to express $J(k,m,C,\omega)$ in terms of the effective mass as the only unknown parameter as follows

$$J(m,\omega) = \frac{\sqrt{\left(1 - \frac{m\omega^2}{k}\right)^2 + \frac{4mD^2\omega^2}{k}}}{\sqrt{\left(1 - \frac{m\omega_M^2}{k}\right)^2 + \frac{4mD^2\omega_M^2}{k}}} \quad (2.13)$$

The only unknown independent variable in $J(m,\omega)$ is now the effective mass. By sweeping through a series of values of m and calculating numerical values of $J(m,\omega)$, it is possible to determine the specific value of m for which $J(m,\omega)$ is equal to the experimental value of the J ratio

$$J(m,\omega) = J(\omega) \quad (2.14)$$

This condition determines the value of the effective mass required by the Kelvin model to fit the experimentally measured dynamic frequency response curve. Placing this calculated value of the effective mass into Equations 2.8, 2.11, and 2.12 gives the proper values of k and C required to fit the experimental frequency response data. The necessary computer programs to accomplish this work on a digital computer have been developed and will be referred to as the WES Dynamic Frequency Response Program, ECNST (Appendix A).

The results in Equations 2.6-2.13 are valid only for a linear theory of pavement response to a constant (frequency-independent) dynamic load and are obtained from the condition $\partial A / \partial \omega = -F_D S^{-2} \partial S / \partial \omega = 0$, or $\partial S / \partial \omega = 0$, where S is given by Equation 2.5. For a nonlinear theory of the frequency response spectrum, the resonance condition is much more complicated and the resonance frequency turns out to be a function of the dynamic load. This is explained in more detail in Section 3.2.2.

2.4 DETERMINATION OF SUBGRADE MODULUS BY FREQUENCY RESPONSE METHOD

The value of the spring constant that is determined from the measured frequency response spectrum will be used to determine the subgrade modulus. The theory of the linear elastic layered half-space predicts a theoretical value of the static spring constant k_T which depends on the radius of the loaded area and on the elastic moduli of the subgrade and the pavement layers. Computer programs are available which calculate the value of k_T if the Young's modulus and Poisson's ratio of each layer of the half-space are known. A well-known computer program of this kind is the Chevron Program. The procedure for determining the Young's modulus E_s of the subgrade is shown in Figure 4. The measured values of f_M , A_M , and $J(\omega)$ are inserted into the WES Dynamic Frequency Response Program and values of k , m , and C are determined. The Young's modulus and Poisson's ratio of the layers of the pavement are selected and entered into the Chevron Program. The subgrade modulus E_s is then iterated in the Chevron Program and a series of values of

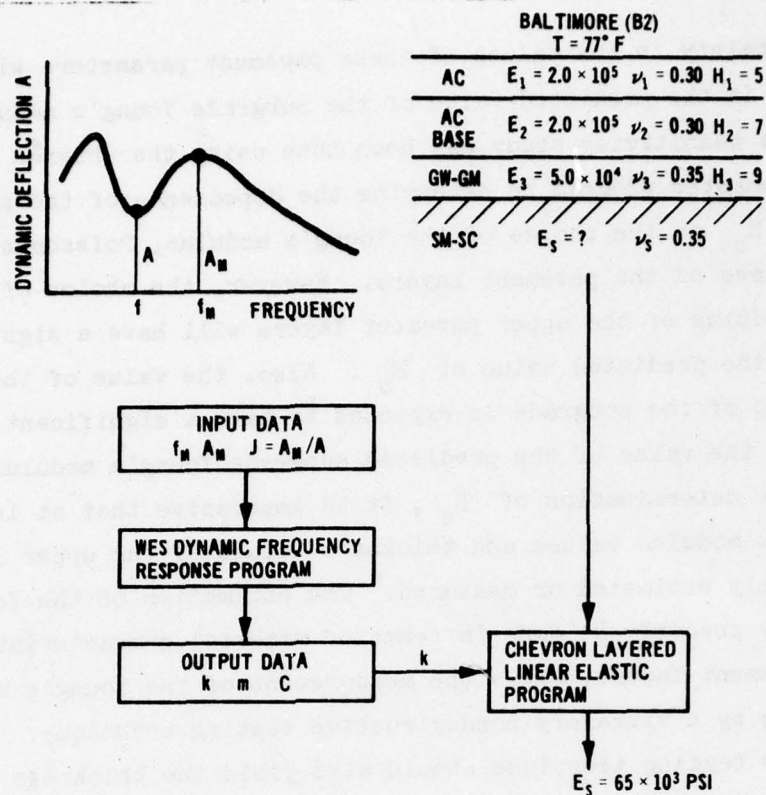


Figure 4. Method of calculating subgrade Young's modulus from measured frequency response curves

k_T is determined. The proper value of E_s is determined by the condition

$$k = k_T \quad (2.15)$$

where k_T is the theoretical value of the spring constant based on layered linear elastic theory. The predicted value of E_s will depend on the values of the elastic moduli selected for the pavement layers.

The value of E_s that is predicted by the WES Dynamic Frequency Response Program depends on the choice of the values of the Young's modulus and Poisson's ratio of each pavement layer and also on the choice of Poisson's ratio of the subgrade soil. The predicted value of E_s also depends on the thickness of each pavement layer.

Any uncertainty in the values of these pavement parameters will lead to errors in the predicted value of the subgrade Young's modulus.

No sensitivity study has been done using the Chevron layered elastic computer program to determine the dependence of the predicted value of E_s on the choice of the Young's modulus, Poisson's ratio, and thickness of the pavement layers. However, the choice of the Young's modulus of the upper pavement layers will have a significant effect on the predicted value of E_s . Also, the value of the Poisson's ratio of the subgrade is expected to have a significant influence on the value of the predicted subgrade Young's modulus. For an accurate determination of E_s , it is imperative that at least the Young's modulus values and thickness values of the upper layers be accurately estimated or measured. The estimation of the Young's modulus may possibly be done in terms of material characteristics of the pavement layers, while the measurement of the Young's modulus may be done by a vibratory nondestructive testing technique. The non-destructive testing technique should also yield the thickness of each pavement layer.

In this report the values of the Young's modulus of the base, subbase, and subgrade materials were estimated by using the Shell formula $E = 1500 \text{ CBR}$. The Shell formula is a straight-line fit through a set of data points and is at best an approximation.^{7,8} The values of the Young's modulus of the AC wearing surface are temperature-dependent and were estimated from an Asphalt Institute curve that is discussed in Section 3.4. The Young's modulus for PCC was taken to be 4×10^6 psi. The values of the Young's moduli of the pavement layers are essentially estimated values and do not represent measured values obtained from laboratory tests on undisturbed samples of pavement materials. The CBR values were measured at the time of the pavement construction, and these values may have changed somewhat by the time that the vibratory nondestructive field tests were performed. The Shell formula gives only approximate values of the Young's modulus.

The subgrade Young's modulus predicted by the Shell equation $E_s = 1500 \text{ CBR}$ is obtained by wave propagation techniques and refers to

the Young's modulus of the subgrade at a confining pressure equal to the overburden pressure due to the pavement layers above. The Shell equation does not include the effects of the dynamic and static loads generated by the vibrator, because the wave propagation measurements are done at the pavement surface a considerable distance away from the vibrator. At this distance the static stress and strain in the pavement and subgrade due to the static weight of the vibrator is practically zero. At this large distance the amplitude of the elastic waves is small so that it is the linear elastic Young's modulus that is measured.

Poisson's ratio for PCC is taken to be 0.2. The value of Poisson's ratio was taken to be 0.3 for AC and bituminous base course materials at all temperatures. A value of 0.35 was assigned to all other base, subbase, and subgrade materials. These values of Poisson's ratio are simple estimates and are not based on laboratory tests done on undisturbed samples taken from the specific pavement sites that were investigated. However, these values of Poisson's ratio were used consistently for all of the pavement sites that were investigated. The layer thicknesses were assumed to be those obtained from construction specifications. In general, no measurements of layer thickness were made. This procedure gives the pavement structure for the example that is used in Figure 4. It is imperative that the elastic moduli and thickness of each pavement layer be known accurately for the prediction of the subgrade Young's modulus.

In principle, it is possible to obtain Poisson's ratio as well as Young's modulus from frequency response measurements performed directly on a subgrade. The theory of a dynamic loading on a linear elastic half-space gives a simple connection between the subgrade elastic constants ν_s and E_s and the theoretical values of the spring constant and damping constant for a homogeneous elastic half-space. This theory gives the following theoretical expressions for the spring constant and the damping constant, respectively:

$$k_H = \frac{4G_s a}{1 - \nu_s} \quad (2.16)$$

$$C_H = \frac{3 \cdot 4a^2}{1 - \nu_s} \sqrt{\frac{G_s \gamma_s}{g}} \quad (2.17)$$

where

k_H = theoretical value of the spring constant

G_s = shear modulus of the elastic half-space

a = radius of vibrator baseplate

ν_s = Poisson's ratio of the elastic half-space

C_H = theoretical value of the damping constant

γ_s = weight density of elastic half-space

g = acceleration of gravity

From Equations 2.16 and 2.17, it follows that Poisson's ratio and Young's modulus of the elastic half-space are given by

$$\nu_s = 1 - B \quad (2.18)$$

$$E_s = \frac{B(2 - B)k_H}{2a} \quad (2.19)$$

where B is a dimensionless number given by

$$B = \frac{(3 \cdot 4)^2 a^3 \gamma_s k_H}{4gC_H^2} \quad (2.20)$$

Equations 2.18-2.20 are derived from the assumption of a linear elastic half-space, and the assumption that all of the damping of the vibrating source on the surface of the half-space is due to the mechanical radiation moving to infinity in the half-space. The results in Equations 2.18-2.20 are applied by assuming that $C = C_H$ and $k = k_H$; i.e., it is assumed that the experimental values of k and C obtained from the frequency response spectrum that is measured from tests done directly on a subgrade are equal to their corresponding theoretical values for a homogeneous elastic half-space. Experimental tests show that this is a poor assumption and Equations 2.18 and 2.19 give poor results.

2.4.1 NUMERICAL RESULTS OF FREQUENCY RESPONSE METHOD

Values of k , m , C , and E_s have been obtained for several airport pavement sites and are listed in Table 1. This table lists the sites according to increasing values of the dynamic stiffness modulus (DSM), which is the slope of the dynamic load-deflection curves at a dynamic load of 14 kips. It is seen that the measured spring constant k increases with increasing pavement strength and that k is not equal to the DSM value. The effective mass is presented as a ratio to the above-surface (vibrator) mass and increases with the strength of the pavement. The effective mass is not equal to the above-surface mass, and any theory which a priori assumes that $m = m_v$ cannot be used to fit the experimental frequency response data. The value of the damping constant also increases with increasing pavement strength. Table 1 shows the results for AC pavements, but similar results are expected for rigid pavements. The predicted values of E_s are compared to those modulus values that are predicted by the Shell method ($E_s = 1500 \text{ CBR}$).^{7,8} The values of E_s predicted by the combined WES Frequency Response Program and the Chevron Program are three to five times larger than those predicted by the CBR method.

There are several possible reasons for the discrepancy in the values of E_s predicted by these two methods:

- a. The pavement-subgrade system is nonlinear under dynamic and static loading, and the predicted value of the subgrade Young's modulus includes the effects of the dynamic and static loads generated by the vibrator.
- b. The subgrade is not uniform and the theoretical layered elastic half-space model may require a rigid boundary below the subgrade.
- c. Reflections from a lower boundary layer add to the motion of the pavement surface.
- d. The relationship $E_s = 1500 \text{ CBR}$ is only an approximation and refers to a static elastic Young's modulus corresponding to the static overburden pressure in the subgrade.

When a rigid boundary such as bedrock is present relatively close to the

Table 1

Numerical Results for Frequency Response MethodApplied to AC Pavements

Location	D_s kips/in.	m/m_v	$C, 10^4 \text{ lb}$ $\times \text{sec/in.}$	k kips/in.	E_s (Chevron) 10^3 psi	E_s (CBR) 10^3 psi
B2	700	1.7	1.0	2137	65	21
N18	770	2.0	0.8	1500	58	27
W1	860	1.8	0.4	2620	136	30
B3	1630	2.0	1.1	2140	35	25
W2	1940	2.4	1.3	2470	69	30
P14	2120	2.5	1.5	2610	139	30
P13	2780	4.4	2.0	3500	153	30
B1	3120	10.0	2.8	4270	140	21

pavement surface, it is possible that the effects listed in b and c may be of importance for determining the motion of a pavement surface that is subjected to a sinusoidal dynamic loading. However, the discrepancy between the values of E_s predicted by the Shell method and those predicted by the frequency response spectra method also occurs in cases where the subgrade is relatively uniform and contains no obvious discontinuities. It is likely that the discrepancy in the values of E_s determined by these two methods is due to a combination of the effects listed in a, b, c, and d.

A basic difficulty with the layered linear elastic interpretation of the frequency response spectra is that the Young's modulus of the subgrade that is obtained by this method includes the effects of the magnitude of the static and dynamic loads of the vibrator that was used to determine the frequency response spectra. It will be shown in Section 3 that for the case of subgrade soils that behave nonlinearly under static and dynamic loads, the static and dynamic loads of the vibrator precondition the pavement and subgrade so that the value of the spring constant k determined from the frequency response data must necessarily include the effects of the static and dynamic loadings.

Therefore, it is not logical to set $k = k_T$, where k_T is the linear elastic spring constant of the pavement-subgrade system which is related to the static elastic Young's modulus of each pavement layer and to the Young's modulus of the subgrade.

The subgrade Young's modulus predicted from the frequency response spectrum by the method described above should not necessarily agree with the result $E_s = 1500 \text{ CBR}$, because the latter equation refers to the static elastic Young's modulus whose value is independent of the static and dynamic loading of a vibrator and is dependent only on the natural static overburden pressure in the subgrade beneath the pavement. It is important to realize that the value of a measured subgrade Young's modulus depends on the method that is used for its measurement. The Young's modulus appearing in $E_s = 1500 \text{ CBR}$ is obtained by wave propagation methods using small amplitude elastic waves, so that this modulus is a measure of the linear elastic properties of the soil at very low dynamic stress and strain levels and at a static confining pressure equal to the overburden pressure. The Young's modulus obtained from the spring model of the frequency response measurements refers to relatively high values of dynamic and static stress and strain in the subgrade because of the vibrator loading at the pavement surface. Because subgrade soils behave nonlinearly under dynamic and static loadings, it is expected that the magnitude of the static and dynamic loads of the WES 16-kip vibrator will affect the value of the subgrade Young's modulus that is obtained from frequency response spectrum measurements.

The values of the Young's modulus obtained by the Shell formula and the Young's modulus obtained directly from the frequency response spectrum must be affected by the overburden pressure on the soil at the top of the subgrade. The Young's modulus of soils depends on the magnitude of the confining pressure. The confining pressure in the subgrade of a pavement is due to the weight of the pavement materials above it, and this overburden pressure affects the Young's modulus of the soil in the subgrade. A rule of thumb for calculating the overburden pressure due to a pavement is $\sigma_{OB}(\text{psi}) = h(\text{ft})$, where σ_{OB} is the overburden

pressure and h is the depth of the point where the overburden pressure is to be calculated. Thus, at a depth of 2 ft, the overburden pressure is approximately 2 psi. The value of E_s must include the effects of this small overburden pressure; i.e., $E_s = E_s(\sigma_{OB})$.

The frequency response spectrum may eventually yield a good method for determining the Young's modulus of the subgrade, but a nonlinear dynamic pavement response theory will be required to do this. Only by a nonlinear theory can the effects of the magnitude of the static and dynamic load be separated from the predicted value of E_s . A nonlinear theory of pavement response is presented in Section 3.

2.5 POSSIBLE EXPLANATIONS OF MULTIPLE RESONANCE PEAKS IN THE FREQUENCY RESPONSE SPECTRUM

The measured frequency response spectrum generally contains many peaks. Two possible explanations for the existence of these multiple peaks are:

- a. They may represent the reinforcement and annihilation of waves reflected from lower boundary layers.
- b. They may represent the fact that the mass of pavement and soil is a dynamic system with many degrees of freedom and more than one normal mode of motion may be excited simultaneously.

If the reflection hypothesis is valid and the motion of the pavement surface is due in part to reflections from subsurface boundaries, then the frequency response spectrum will not directly be a measure of pavement strength, and the frequency response spectrum method of predicting pavement strength would have to be altered to include the effects of reflections.

If the pavement and subgrade are behaving as a dynamic system with more than one degree of freedom, a multiple-mass linear spring model would have to be used to describe the frequency response spectrum measured at the pavement surface. The multiple-mass models such as the one shown in Figure 3 are difficult to handle because they are very complicated and contain too many parameters to use the simple analytical method of determining the elements of the model as described in

Equations 2.6-2.14. Nevertheless, it should be remembered that the energy put into the pavement-subgrade system by a vibrator is propagated throughout the pavement and subgrade in the form of elastic waves, and therefore a lumped-mass spring model is an extreme idealization of the actual physical situation. Possibly a somewhat more physical model for pavement vibrations would be the vibration of a rod of distributed mass. This model will have a frequency response spectrum with multiple resonance peaks which may possibly fit the measured frequency response spectra.

3. DYNAMIC LOAD-DEFLECTION CURVE METHOD OF DETERMINING THE SUBGRADE MODULUS

3.1 GENERAL CONSIDERATIONS

The layered elastic theoretical approach to the calculation of allowable load or required overlay thickness for a pavement requires the values of the elastic moduli of the subgrade and each layer of the pavement. In the previous section, a linear spring model was used to determine a static spring constant for the pavement-subgrade system, from which a subgrade Young's modulus was obtained by using the linear elastic Chevron computer program to relate the static spring constant to the elastic moduli of the pavement and subgrade. The disagreement between these predicted values of the subgrade Young's modulus and the values of the static elastic Young's modulus of the subgrade given by the equation $E_s = 1500 \text{ CBR}$ suggests that the Young's modulus predicted from frequency response spectra includes the nonlinear effects of the static and dynamic loads exerted by the vibrator on the pavement surface. The basic nonlinearity of the pavement and subgrade must be considered if a correlation is to be made between the impedance method of vibratory nondestructive pavement testing and the wave propagation method of non-destructive pavement testing.

An alternative method for determining the subgrade Young's modulus from vibratory nondestructive test data is the use of the dynamic load-deflection curves measured at the pavement surface for a fixed frequency and a fixed static load. These dynamic load-deflection curves are generally nonlinear for weak pavements and become more linear for stronger pavements. The values of the Young's modulus of the subgrade depend on the static overburden pressure that exists in the subgrade. For the vibratory nondestructive tests using the 16-kip vibrator, the stress in the subgrade is due to the static and dynamic loads exerted by the vibrator in addition to the static overburden pressure. The quantity of physical interest is the Young's modulus of the subgrade soil at a confining pressure equal to the static overburden pressure. Therefore the extraneous effects of the dynamic and static loads

generated by the vibrator must be removed from the determination of the Young's modulus of the subgrade.

A nonlinear dynamic theory of pavement response is required to extract the static elastic subgrade Young's modulus for a given overburden pressure from the measured dynamic load-deflection curves. It is important to extract the Young's modulus of the subgrade because it is the Young's modulus that appears in the relation $E_s = 1500 \text{ CBR}$ that is derived from elastic wave propagation. Elastic wave propagation experiments on soils are done with small amplitudes so that linear elastic Young's moduli are obtained from these experiments. If the nonlinear (large amplitude) load-deflection tests are to agree with the wave propagation tests, a Young's modulus must be extracted from the nonlinear load-deflection test data.

Over the years WES has collected an extensive set of dynamic load-deflection curves that have been obtained on many airfield pavements throughout the country. A typical measured dynamic load-deflection curve appears in Figure 5, and it is seen that this curve is

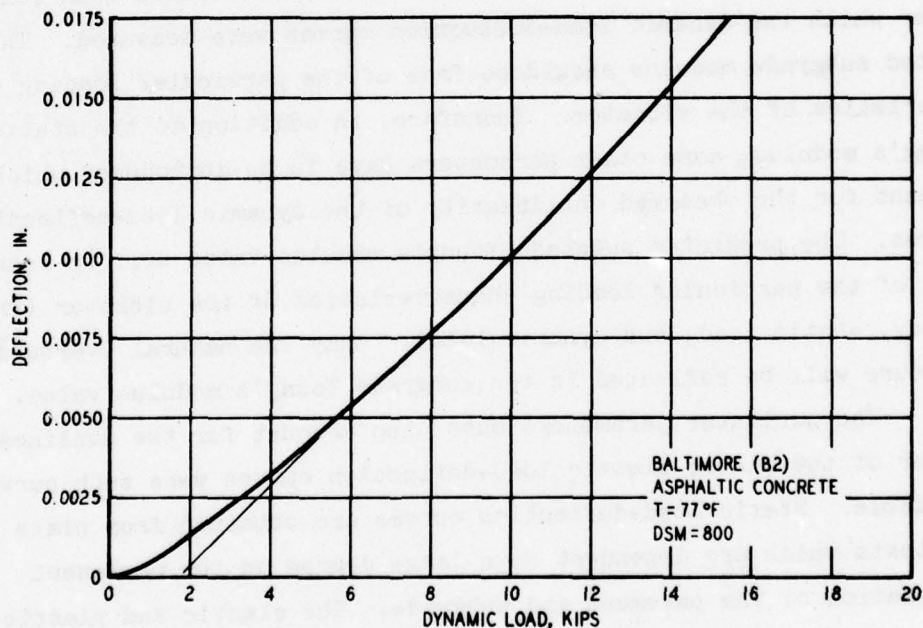


Figure 5. Typical dynamic load-deflection curve

generally nonlinear. The dynamic load-deflection curves are measured at the surface of a pavement. This section develops the nonlinear dynamic theory of the dynamic load-deflection curves which is necessary for the determination of the Young's modulus of the subgrade. The nonlinear dynamic theory predicts the response of a pavement to a dynamic load. This theoretical model will depend on the Young's moduli of the pavement layers and subgrade and on model parameters that describe the inertial, damping, and nonlinear elastic properties of the pavement and subgrade. The subgrade Young's modulus value is predicted by requiring the dynamic load-deflection curves predicted by the nonlinear dynamic theory to agree with the measured dynamic load-deflection curves.

3.2 NONLINEAR THEORY OF PAVEMENT RESPONSE TO DYNAMIC SURFACE LOADINGS

The nonlinear dynamic load-deflection curves were measured by sweeping through a range of dynamic loads up to 15 kips for a frequency of 15 Hz and at a static surface loading of 16 kips. The nonlinear dynamic theory must account for the frequency and static load conditions under which the dynamic load-deflection curves were measured. The predicted subgrade modulus should be free of the particular loading characteristics of the vibrator. Therefore, in addition to the static Young's modulus, some other parameters have to be introduced which will account for the observed nonlinearity of the dynamic load-deflection curves. The predicted subgrade Young's modulus value will be independent of the particular loading characteristics of the vibrator (frequency, static load, and dynamic load). Only the natural overburden pressure will be reflected in the subgrade Young's modulus value.

The nonlinear parameters must also account for the nonlinear behavior of the static elastic load-deflection curves were such curves available. Static load-deflection curves are obtained from plate bearing tests which are dependent to a large degree on the permanent deformation of the pavement and subgrade. The elastic and plastic deformations must be separated in order to obtain the static elastic load-deflection curves. Static elastic load-deflection curves have not

yet been obtained using the WES 16-kip vibrator.

Dynamic tests on pavements and subgrades avoid the problem of plastic flow by operating at a frequency (15 Hz) that is sufficiently high so as to obtain resilient deflections whose values are essentially independent of the plastic flow of the pavement and subgrade materials. Dynamic theories can be developed which describe the resilient response of pavements, and these theories can be extrapolated to the case of zero frequency to obtain the static elastic deflection of the pavement surface. However, the plastic part of the displacement under a static load is generally much larger than the elastic part, and a complete description of pavement performance will require static load tests as well as resilient dynamic tests.

The determination of the Young's modulus of the subgrade from measured dynamic load-deflection curves requires a nonlinear dynamic theory of the elastic response of a pavement to dynamic loads. A nonlinear dynamic theory of pavement response will be used to remove the extraneous effects of the dynamic and static loads generated by the vibrator on the determination of the Young's modulus of a subgrade. The Young's modulus will depend only on the static overburden pressure in the subgrade.

3.2.1 EQUATION OF MOTION OF A NONLINEAR OSCILLATOR

The nonlinear theory of pavement response to a vibratory load assumes that the pavement-subgrade system can be described by a lumped-mass nonlinear oscillator whose equation of motion is written as⁴

$$m\ddot{x} + C\dot{x} + k_{00}x + bx^3 + ex^5 = F_D + F_S \quad (3.1)$$

where:

m = effective mass of the pavement-subgrade system

x = total displacement of the pavement surface beneath the vibrator baseplate

C = damping constant

k_{00} = linear spring constant

b = third-order nonlinear pavement and subgrade parameter

e = fifth-order nonlinear pavement and subgrade parameter

F_D = dynamic load applied to the pavement surface

F_S = static load applied to the pavement surface

The total displacement of the pavement surface is composed of static and dynamic parts as follows

$$x = x_e + \xi \quad (3.2)$$

where

x_e = static elastic deflection of the pavement surface

ξ = dynamic elastic deflection of the pavement surface

Placing Equation 3.2 into Equation 3.1 gives the following equation of motion:

$$m\ddot{\xi} + C\dot{\xi} + (k_{00} + 3bx_e^2 + 5ex_e^4)\xi + b\xi^3 + e\xi^5 + \xi g(x_e, \xi) = F_D \quad (3.3)$$

where

$$g(x_e, \xi) = 3bx_e\xi + 10ex_e^3\xi + 10ex_e^2\xi^2 + 5ex_e\xi^3 \quad (3.4)$$

For convenience in manipulating Equation 3.3, it is necessary to use a time-averaged expression for Equation 3.4 as follows:

$$g(x_e, \xi) = 3a_1bx_e^2 + 5a_2ex_e^4 + a_3b\xi^2 + a_4e\xi^4 \quad (3.5)$$

where a_1 , a_2 , a_3 , and a_4 are coefficients to be determined from the measured dynamic load-deflection data. Combining Equations 3.3 and 3.5 gives the motion equation as

$$m\ddot{\xi} + C\dot{\xi} + k_0\xi + b_0\xi^3 + e_0\xi^5 = F_D \quad (3.6)$$

where

$$k_0 = k_{00} + 3be_2x_e^2 + 5ee_4x_e^4 \quad (3.7)$$

$$\theta = 1 + a_3 \quad (3.8)$$

$$\eta = 1 + a_4 \quad (3.9)$$

$$\epsilon_2 = 1 + a_1 \quad (3.10)$$

$$\epsilon_4 = 1 + a_2 \quad (3.11)$$

A mechanical vibrator operating on the surface of a pavement produces static and dynamic deflections of the pavement surface. For a linear dynamic system, the static and dynamic deflections are independent, and the static displacement of a pavement surface would not appear in the dynamic equation of motion of the pavement surface.⁴ For a nonlinear dynamic system, such as an actual pavement or subgrade, the static displacement of the pavement surface appears in the dynamic equation of motion of the pavement surface, so that the static and dynamic displacements are not independent and must be calculated jointly from the dynamic equations of motion.⁴

The parameters θ , η , ϵ_2 , and ϵ_4 appearing in Equations 3.6-3.11 represent a simple approximate way of treating the interdependence of the static and dynamic displacements of the pavement surface. These parameters describe the approximation of writing Equation 3.4 in the form of Equation 3.5 which brings the equation of motion into the solvable form of Equation 3.6. The parameters θ , η , ϵ_2 , and ϵ_4 depend on the pavement strength and are determined by requiring Equation 3.6 to adequately describe the dynamic load-deflection curves. These four parameters describe the higher order interaction terms of ξ and ϵ_e and their departure from the value unity indicates the degree of mixing of the static deflection and dynamic deflection terms that occur in Equation 3.6.

The nonlinear parameters b and e determine the static load-deflection curves, as can be seen from Equation 3.1 which for the static case becomes⁴

$$F_S = k_{00}x_e + bx_e^3 + ex_e^5 \quad (3.12)$$

In general, it is found that $b < 0$ and $e > 0$ for pavements and most subgrades.

3.2.2 THEORY OF DYNAMIC LOAD- DEFLECTION CURVES AND THE FREQUENCY RESPONSE SPECTRUM

The problem remains to solve the nonlinear Equation 3.6. This can be done by casting Equation 3.6 into an equivalent linear form for which the dynamic amplitude is given by

$$\xi = F_D / S \quad (3.13)$$

where

$$S = \sqrt{(k - m\omega^2)^2 + C^2\omega^2} \quad (3.14)$$

where

- S = dynamic stiffness
- k = dynamic spring constant
- m = effective mass
- ω = angular frequency
- C = damping constant

The requirement that Equations 3.13 and 3.14 be a solution of Equation 3.6 is that the spring constant in Equation 3.14 is given by⁴

$$k = k_0 + \frac{3}{4} b \theta \xi^2 + \frac{5}{8} e \eta \xi^4 \quad (3.15)$$

Therefore, the spring constant for a nonlinear system depends on the dynamic and static displacements of the pavement surface.

The conclusion that the spring constant for the equivalent linear form of a nonlinear pavement-subgrade system depends on the dynamic and static elastic deflections of the pavement surface, as shown in Equation 3.15, explains the difficulties that were encountered in predicting the subgrade modulus from the frequency response spectrum method (Section 2.4.1). The value of k that is obtained by applying Equations 2.4 and 2.5 (or equivalently Equations 3.13 and 3.14) to the

frequency response spectrum of a nonlinear pavement and subgrade is now seen to include the effects of the static load and the dynamic load through the nonlinear terms appearing in Equations 3.15 and 3.7.

Therefore the value of k obtained directly from the frequency response spectrum is actually not the linear elastic spring constant of the pavement and subgrade, and this value of k cannot be used in the Chevron layered linear elastic computer program to determine the value of the Young's modulus of the subgrade. It is actually k_{00} which is the linear elastic spring constant, and it is k_{00} that is directly related to the elastic constants of the pavement and subgrade. A nonlinear dynamic theory of pavement response is required to separate the value of k into its component parts and extract the value of k_{00} from which the subgrade modulus can be determined.

Placing Equation 3.15 into Equations 3.13 and 3.14 and solving for the dynamic amplitude yields the result⁴

$$\xi = \frac{F_D}{S_0} (1 + \alpha_1 \psi + \alpha_2 \psi^2 + \dots) \quad (3.16)$$

where

$$S_0 = \sqrt{(k_0 - m\omega^2)^2 + c^2\omega^2} \quad (3.17)$$

$$\psi = \frac{F_D^2}{S_0^4} \quad (3.18)$$

$$\alpha_1 = -\frac{3}{4} b\theta(k_0 - m\omega^2) \quad (3.19)$$

$$\alpha_2 = \frac{7}{2} \left(\frac{3}{4}\right)^2 b^2\theta^2(k_0 - m\omega^2)^2 - S_0^2 \left[\frac{5}{8} n\theta(k_0 - m\omega^2) + \frac{1}{2} \left(\frac{3}{4}\right)^2 b^2\theta^2 \right] \quad (3.20)$$

The result in Equation 3.16 gives the dynamic deflection of the pavement surface directly beneath the vibrator baseplate in terms of the dynamic load applied to the pavement surface by the vibrator baseplate. Equations 3.16 and 3.18 show that the dynamic deflection is a nonlinear function of the dynamic load. The nonlinear portions of the dynamic load-deflection curves are described by the terms $\alpha_1 \psi$ and $\alpha_2 \psi^2$ in

Equation 3.16, while the linear portion of these curves, which occurs at low dynamic loads, is given by F_D/S_0 . As shown by Equations 3.16-3.20, the coefficients S_0 , α_1 , and α_2 depend on the operating frequency of the vibrator and on the structure of the pavement and subgrade.

Two types of dynamic elastic modulus can be obtained from the measured dynamic load-deflection curves. The secant modulus (impedance) is obtained from Equations 3.13 and 3.16 and is

$$S = S_0 \left(1 + \beta_1 \psi + \beta_2 \psi^2 \right) \quad (3.21)$$

where

$$\begin{aligned} \beta_1 &= -\alpha_1 \\ \beta_2 &= \alpha_1^2 - \alpha_2 \end{aligned}$$

The tangent modulus is the slope of the load-deflection curve or the DSM. It is given by

$$DSM = \frac{dF_D}{d\xi} = \left(\frac{d\xi}{dF_D} \right)^{-1} \quad (3.22)$$

From Equation 3.16, it follows that

$$DSM = S_0 \left(1 + \delta_1 \psi + \delta_2 \psi^2 \right) \quad (3.23)$$

where

$$\begin{aligned} \delta_1 &= -3\alpha_1 \\ \delta_2 &= 9\alpha_1^2 - 5\alpha_2 \end{aligned}$$

Numerical values of the DSM are generally calculated for $F_D = 15$ kips.

The coefficients S_0 , α_1 , and α_2 can be obtained by fitting the mathematical expression in Equation 3.16 to the dynamic load-deflection curves measured for a specific frequency which is usually 15 Hz for the WES 16-kip vibrator. The coefficient S_0 is the slope of the load-deflection curve at the condition $F_D = 0$, while the coefficients α_1 and α_2 represent the curvature of the dynamic

load-deflection curves. The three coefficients S_0 , α_1 , and α_2 are obtained by fitting a polynomial form containing linear, cubic, and fifth-order terms to each measured dynamic load-deflection curve. A computer program called NLIN was developed in part to accomplish this task (Appendix B).

The coefficients k_0 , m , C , k_{00} , b , e , θ , η , ϵ_2 , and ϵ_4 that appear in Equations 3.7 and 3.17-3.20 are obtained jointly from S_0 , α_1 , and α_2 by examining many measured dynamic load-deflection curves obtained at pavement sites of known structure, and finding the combination of parameters which describes these curves and produces theoretical values of the static elastic deflection of the pavement surface which are comparable to the values of the dynamic deflection of the pavement surface (Appendix B).

The measured dynamic load-deflection curves of medium strength AC pavements are generally nonlinear when measured at a fixed frequency of 15 Hz. They are even more nonlinear when measured at other frequencies. The shape of the measured dynamic load-deflection curves depends as much on the frequency of operation of the vibrator as on the structure of the pavement and subgrade.⁴ Experience gained from years of nondestructive testing of pavements indicated that operating the WES 16-kip vibrator at a frequency of 15 Hz produced dynamic load-deflection curves which were generally more smooth than those measured at other frequencies. Load sweep tests conducted at frequencies of 5, 10, 20, and 25 Hz produced dynamic load-deflection curves that were more curved than those obtained at a frequency of 15 Hz.⁴ Therefore, there is a good practical reason for conducting the load sweep vibratory tests at a frequency of 15 Hz.

There is also a good theoretical reason for the straightening effect of the 15-Hz operating frequency of the vibrator. As seen from Equations 3.16-3.19, the degree of nonlinearity of a dynamic load-deflection curve depends on the strength of the pavement and the frequency of operation of the vibrator. The strength of the pavement affects the degree of nonlinearity of the dynamic load-deflection curves through the term S_0^{-4} that appears in Equations 3.16 and 3.18. The S_0^{-4} term

shows that strong pavements tend to be more linear than weak pavements. From Equation 3.19 it is clear that there is a critical frequency for which the first-order nonlinear term vanishes and this frequency is given by

$$f_c = \frac{1}{2\pi} \sqrt{\frac{k_0}{m}} \quad (3.24)$$

This critical frequency is found to be about 15 Hz for most AC and PCC pavements. At this frequency, the dynamic load-deflection curves should become less curved in the regions of low dynamic force if the second-order nonlinear term is comparatively small. The straightening effect at the critical frequency will not be strongly evident if the second-order nonlinear term is comparatively large. The resonance frequency f_R has a value close to the critical frequency f_c .

Aside from the few experimental measurements of the critical frequency that are presented in Reference 4, there have been no direct measurements of the critical frequency for different types of pavements and subgrades. The values of the critical frequency that are determined in this report use Equation 3.24 where the parameters k_0 and m were determined by analyzing dynamic load-deflection curves that were measured at a frequency of 15 Hz.

For the nonlinear dynamic theory, Equation 3.16 describes the frequency response spectrum as well as the dynamic load-deflection curves. As was the case for the linear elastic dynamic theory of the frequency response spectrum, an expression is required for the peak frequency and the peak amplitude in terms of the inertial, damping, and elastic parameters of the dynamic model. The model parameters could then be obtained directly from the measured values of the peak frequency and peak amplitude as described for the linear elastic model in Section 2.4.

For the linear elastic dynamic model with a frequency independent dynamic load, the results in Equations 2.6 and 2.7 are derived from the peak condition $\partial A / \partial \omega = -F_D S^{-2} \partial S / \partial \omega = 0$ or equivalently $\partial S / \partial \omega = 0$. For the nonlinear model, the peak condition for the case of a frequency independent dynamic load (constant F_D) can be obtained from Equation 3.16 to be

$$\begin{aligned} \frac{\partial \xi}{\partial \omega} = & -F_D S_0^{-2} \frac{\partial S_0}{\partial \omega} (1 + \alpha_1 \psi + \alpha_2 \psi^2) \\ & + F_D S_0^{-1} \left[\frac{\partial \alpha_1}{\partial \omega} \psi + \frac{\partial \alpha_2}{\partial \omega} \psi^2 + (\alpha_1 + 2\alpha_2 \psi) \frac{\partial \psi}{\partial \omega} \right] = 0 \end{aligned} \quad (3.25)$$

where the frequency dependent functions S_0 , α_1 , α_2 , and ψ are given by Equations 3.17-3.20. Equation 3.25 reduces to the linear elastic peak condition for the linear elastic case which has $\alpha_1 = \alpha_2 = 0$. The solution of Equation 3.25 gives the peak frequency f_M as a function of the dynamic and static loads, inertial mass, damping constant, and the linear and nonlinear elastic pavement parameters; i.e., $f_M = f_M(F_D, F_S, m, C, k_{00}, b, e, \theta, \eta, \epsilon_2, \epsilon_4, \kappa, \ell_0, \ell_2, \ell_4)$. The peak amplitude is found by placing this value of f_M into Equation 3.16. The solution of Equation 3.25 is still under development.

The nonlinear dynamic theory of pavement response predicts the resonance frequency and the peak frequency to be functions of the dynamic and static loads developed by the vibrator. Therefore frequency response curves measured for a series of fixed dynamic loads would be necessary to fit the nonlinear dynamic frequency response theory to the experimental data. Several frequency response curves have been measured at pavement sites for a series of fixed dynamic loads, and these data indicate that the frequency response spectrum changes shape considerably for different dynamic force levels. It may not be an easy matter to detect a dependence of the resonance frequency on the magnitude of the dynamic load. Further experimental tests are necessary.

When developed, the nonlinear theory of the frequency response spectrum will be used to determine the subgrade Young's modulus in a manner similar to that described for the linear elastic dynamic theory described in Section 2.4. The nonlinear theory of the frequency response spectrum will eliminate the effect of the static and dynamic loads from the predicted value of the subgrade Young's modulus. The nonlinear dynamic theory shows that the measured resonance frequency and peak frequency will depend on the magnitudes of the static and dynamic loads developed by the vibrator, and so it is clear why

Equation 2.11 of the linear elastic theory predicts values of k , and also E_s , that are much larger than the values given by $E_s = 1500 \text{ CBR}$. Equation 2.11 combined with the measured values of f_M produces a spring constant k that depends on the magnitudes of the static and dynamic loads exerted by the vibrator on the pavement surface. Therefore the condition $k = k_T$, where k_T is the linear elastic spring constant given a layered linear elastic theory, cannot be used to determine the value of the subgrade Young's modulus as was done in Section 2.4. A nonlinear dynamic theory is required to remove the extraneous effects of F_D and F_s on the predicted value of the subgrade Young's modulus.

3.2.3 DYNAMIC NATURE OF THE SPRING CONSTANT k

The measurement of the dynamic load-deflection curves determine the linear and nonlinear elastic parameters of a pavement system (k_{00} , $b, e, \theta, n, \epsilon_2, \epsilon_4$). These parameters relate the spring constants k and k_0 . Equation 3.15 shows that the spring constant k that is determined from a dynamic analysis of the nonlinear properties of a pavement-subgrade system is dependent on the dynamic and static displacements of the pavement surface as well as on the elastic constants of the pavement-subgrade system. Therefore, the spring constant k that is determined from the dynamic response of a nonlinear pavement system is a dynamic quantity that is not analogous to an ordinary static spring constant. The theoretical static spring constant determined from a static linear elastic program such as the Chevron Program will depend only on the elastic constants of the pavement. Therefore, the value of k determined from the dynamic response data of a nonlinear pavement cannot logically be compared to the static k_T value determined from static layered elastic computer programs. Static plate bearing tests will result in a spring constant which will also not be directly comparable to the spring constant determined from an analysis of dynamic data.

The dynamic spring constant k is related to the static elastic spring constants k_{00} and k_0 by Equations 3.7 and 3.15. The static elastic spring constant k_0 includes the effects of the nonlinear nature of pavements through terms dependent on the static elastic deflection of

the pavement surface. Therefore k and k_0 must be comparable in magnitude. The spring constant k_{00} is the linear elastic spring constant of the pavement-subgrade system. It is the linear elastic spring constant k_{00} that should agree with the theoretical static elastic spring constant that is given by the Chevron and Shell linear layered elastic computer programs. In any case, the elastic properties of pavements are described by three spring constants k , k_0 , and k_{00} , and the linear spring constant k_{00} is expected to have a value considerably smaller than the values of k and k_0 . Because k and k_0 have comparable magnitudes, the value of resonance frequency will be approximately equal to the value of the critical frequency, $f_R \sim f_c$. The peak frequency f_M will be only slightly smaller than f_R and f_c .

3.2.4 FINITE DEPTH OF INFLUENCE

The nonlinearity of the static elastic and the dynamic elastic load-deflection curves can be related to the assumption that the static elastic stress and strain does not extend to infinite depth in the subgrade but has a finite depth of influence.⁴ The basic nonlinear elastic nature of subgrade materials manifests itself in a finite range for the static stress and strain fields.

The static linear k_{00} and nonlinear parameters b and e can be related to the elastic moduli of the pavement layers and to the depth of influence of the static stress-strain field.⁴ The finite depth of influence is written in terms of the static deflection of the pavement surface as⁴

$$\ell = \ell_0 + \ell_2 x_e^2 + \ell_4 x_e^4 \quad (3.26)$$

where

ℓ = finite depth of influence

ℓ_0, ℓ_2, ℓ_4 = coefficients of the power series expansion of the finite depth of influence.

For the simplest case of a vibrator placed on the surface of a subgrade, the static parameters are⁴

$$k_{00} = \frac{2\pi a^2 \psi (1 - \nu_s) G_s}{\ell_0 (1 - 2\nu_s)} \quad (3.27)$$

$$b = - \frac{4\pi a^2 \Psi l_2 (1 - \nu_s) G_s}{l_0^2 (1 - 2\nu_s)} \quad (3.28)$$

$$e = \frac{6\pi a^2 \Psi \delta (1 - \nu_s) G_s}{l_0 (1 - 2\nu_s)} \quad (3.29)$$

where

$$\delta = \left(\frac{l_2}{l_0} \right)^2 - \frac{l_4}{l_0} \quad (3.30)$$

where

a = radius of vibrator baseplate

ν_s = Poisson's ratio of subgrade

G_s = shear modulus of subgrade

Ψ = volume factor for the frustum of the cone of stress and strain

The expressions for k_{00} , b , and e given by Equations 3.26-3.30 (and their generalization to the case of a pavement over a subgrade) are derived by calculating the work done during the static elastic deflection of a pavement surface due to a static load described by Equation 3.12, and then setting this work equal to the elastic strain energy of the pavement and subgrade.⁴

The volume factor Ψ depends on a parameter κ which gives a measure of the lateral spreading of the static stress and strain in the pavement and subgrade.⁴ The static stress and strain distribution in the pavement and subgrade beneath the static 16-kip load of the WES vibrator is assumed to be confined to a frustum of a cone whose upper radius is equal to that of the vibrator baseplate and whose lower radius (at depth l) is determined by the degree of lateral spreading of the static stress and strain in the pavement and subgrade.⁴ The parameter κ is the ratio of the radius of the lower area of the frustum to that of the upper area.⁴ The radius of the lower circular area of the frustum is κa . Values of κ larger than unity indicate that the static stress and strain spreads in the lateral directions in the pavement and subgrade. The volume factor Ψ is the ratio of the volume of

the frustum of the cone of static stress and strain to the volume of a cylinder whose length is l and whose radius is equal to the radius of the baseplate of the vibrator.⁴

The parameters κ and ψ give a measure of the lateral spreading of the stress and strain in a pavement and subgrade, and indicate how much of the static load applied at the pavement surface is transmitted to the subgrade at a point directly beneath the applied load. An important function of a pavement is the protection of the subgrade from excessive stress and strain which might cause harmful plastic flow of soil in the subgrade. The structure of a pavement is designed in part to protect the subgrade. Therefore, it is important to know the dependence of the lateral stress-strain spreading factor κ on the structure or more simply on the resilient strength of pavements. It is found that the parameter κ is an increasing function of pavement strength.

3.2.5 ELASTIC MODULI OF PAVEMENT LAYERS AND SUBGRADE

The static linear elastic parameter k_{00} and the static nonlinear elastic parameters b and e are related to the structure of the pavement and subgrade because these parameters depend on the elastic moduli of the pavement layers and subgrade. For the case of a vibrator operated directly on a subgrade, the relations in Equations 3.27-3.30 give the theoretical connection between the parameters k_{00} , b , and e and the elastic moduli of the subgrade, E_s and ν_s . For the more general case of a mechanical vibrator operating at the surface of a pavement overlying a subgrade, the parameters k_{00} , b , and e depend on the elastic moduli of the pavement layers as well as on the elastic moduli of the subgrade ($E_1, \nu_1, E_2, \nu_2, \dots, E_s, \nu_s$). For this case, expressions for k_{00} , b , and e in terms of the elastic moduli are presented in Reference 4.

In this way, the theoretical expression for the dynamic stiffness as given by Equations 3.14, 3.7, and 3.15 is connected to the elastic moduli of the subgrade and pavement layers. The predicted value of E_s (assuming $\nu_s = 0.35$) for a subgrade is then obtained by finding

the value of E_s that makes the theoretically predicted dynamic load-deflection curve agree with the dynamic load-deflection curve measured at the surface of a pavement. But, before this can be done, the model parameters must be determined for any pavement location.

3.3 MODEL PARAMETERS

In order to determine the subgrade Young's modulus by vibratory nondestructive tests, all of the parameters that enter into the nonlinear dynamic pavement response model must be known for each pavement test site. This section gives these parameters and describes their dependence on the elastic strength of pavements.

The parameters of the nonlinear pavement response model have been determined empirically because they depend in a complicated manner on the structure of the pavement and subgrade and on the particular vibrator used for nondestructive testing. Each pavement site will have its own characteristic set of parameters when tested with the WES 16-kip vibrator, and these parameters would be different had another vibrator been used. The model parameters were determined at pavement sites of known structure by requiring that the dynamic load-deflection curve predicted theoretically by the nonlinear response model agree with the load-deflection curve determined in the field. By requiring agreement for a number of pavement sites of known structure, it is possible to predict the values for all of these parameters at any pavement site where a dynamic load-deflection curve is measured.

The model parameters m , C , k_{00} , b , e , ℓ_0 , ℓ_2 , ℓ_4 , θ , κ , η , ϵ_2 , and ϵ_4 describe the inertial, damping, and linear and nonlinear elastic properties of the pavement and vibrator system and depend on vibrator characteristics and on the structure of the pavement and subgrade. This dependence is in general very complicated and difficult to determine theoretically. The simplest way to attach the model parameters to the strength of a pavement-subgrade system is to determine these parameters in terms of the measured DSM of a pavement. The DSM value is a suitable choice for a parameter in terms of which to describe the model parameters because it is a measure of the bulk strength of the

pavement and subgrade. The model parameters expressed in terms of the measured DSM correspond to the WES 16-kip vibrator. The vibrator characteristics appear in these parameters because the subgrade Young's modulus to be determined is intended to be independent of the dynamic characteristics of the vibrator. A corresponding set of vibrator parameters will have to be developed for any other vibrator that is to be used for nondestructive testing of pavements.

The theoretical nonlinear dynamic pavement response model was developed to predict the Young's modulus of a subgrade from dynamic load-deflection curves measured at a pavement surface. The nonlinear dynamic model of pavement response gives a theoretical prediction of the DSM of a pavement in terms of the elastic moduli of the pavement layers and subgrade and in terms of the model parameters m , C , k_{00} , b , e , l_0 , l_2 , l_4 , κ , θ , η , ϵ_2 , and ϵ_4 . The nonlinear dynamic model can be used to predict the subgrade Young's modulus only if the elastic moduli of the pavement layers are known and if the model parameters are known as functions of the measured DSM of a pavement. Then, if the DSM is measured at a pavement site, the only unknown parameter in the nonlinear dynamic model is the subgrade Young's modulus. The value of the subgrade Young's modulus can then be varied in the nonlinear dynamic model until the model predicts the measured value of the DSM. This gives the predicted value of the subgrade Young's modulus. Therefore it is important to have the model parameters as known functions of the measured DSM value of a pavement. The model parameters are presented as a function of the measured DSM in Figures 6-21.

The model parameters are determined by applying the nonlinear dynamic theory to the dynamic load-deflection curves measured at pavement sites where the elastic moduli of the pavement layers and subgrade are known. About 30 pavement sites were considered whose DSM values ranged from 300 to 6500 kips/in. Each of the data points in Figures 6-21 represent a pavement location that was studied. The model parameters were determined jointly by a trial and error process for the 30 pavement locations. The model parameters were varied until the parameters

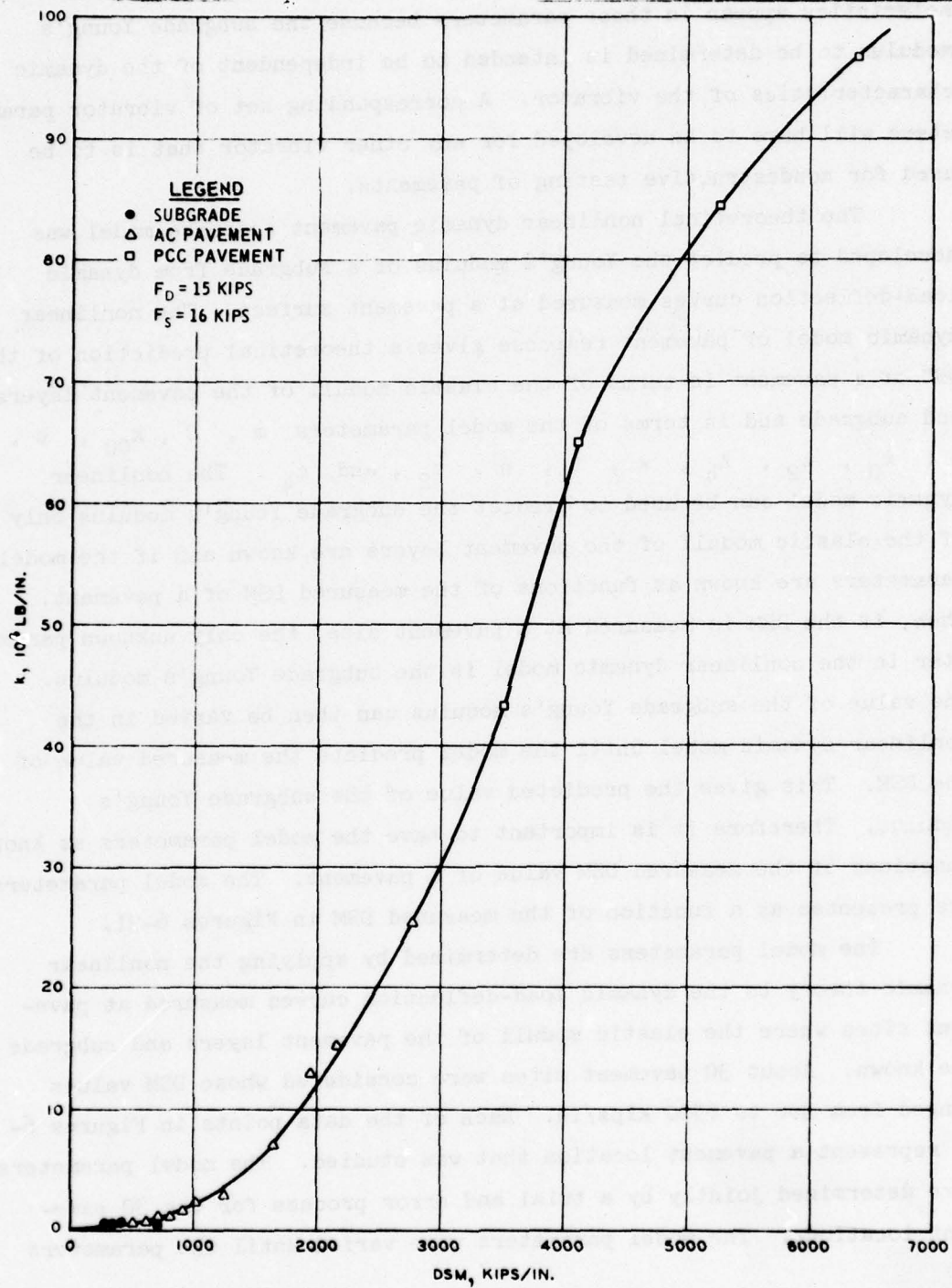


Figure 6. Dynamic spring constant versus measured DSM

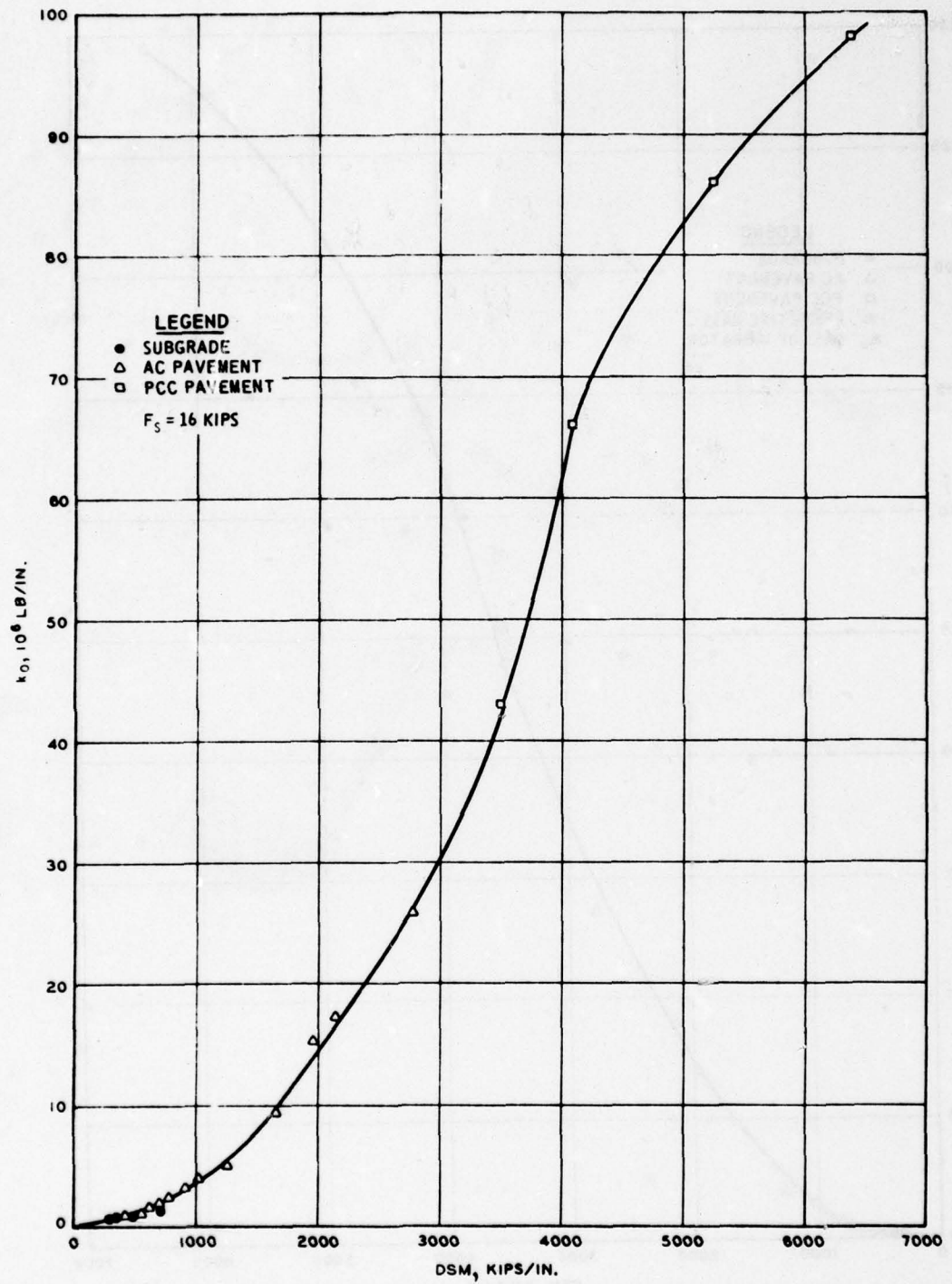


Figure 7. Static spring constant versus measured DSM

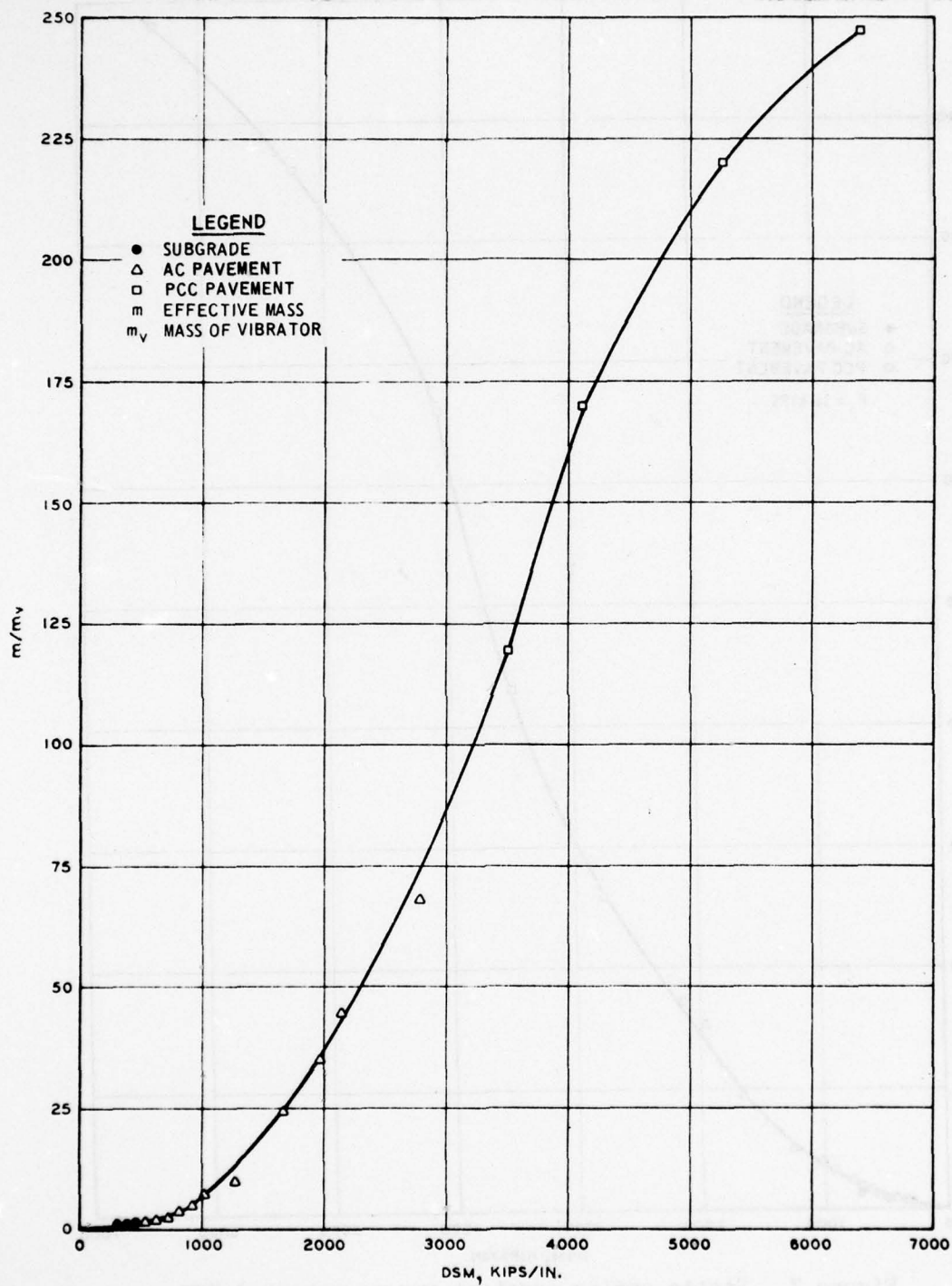


Figure 8. Effective mass ratio versus measured DSM

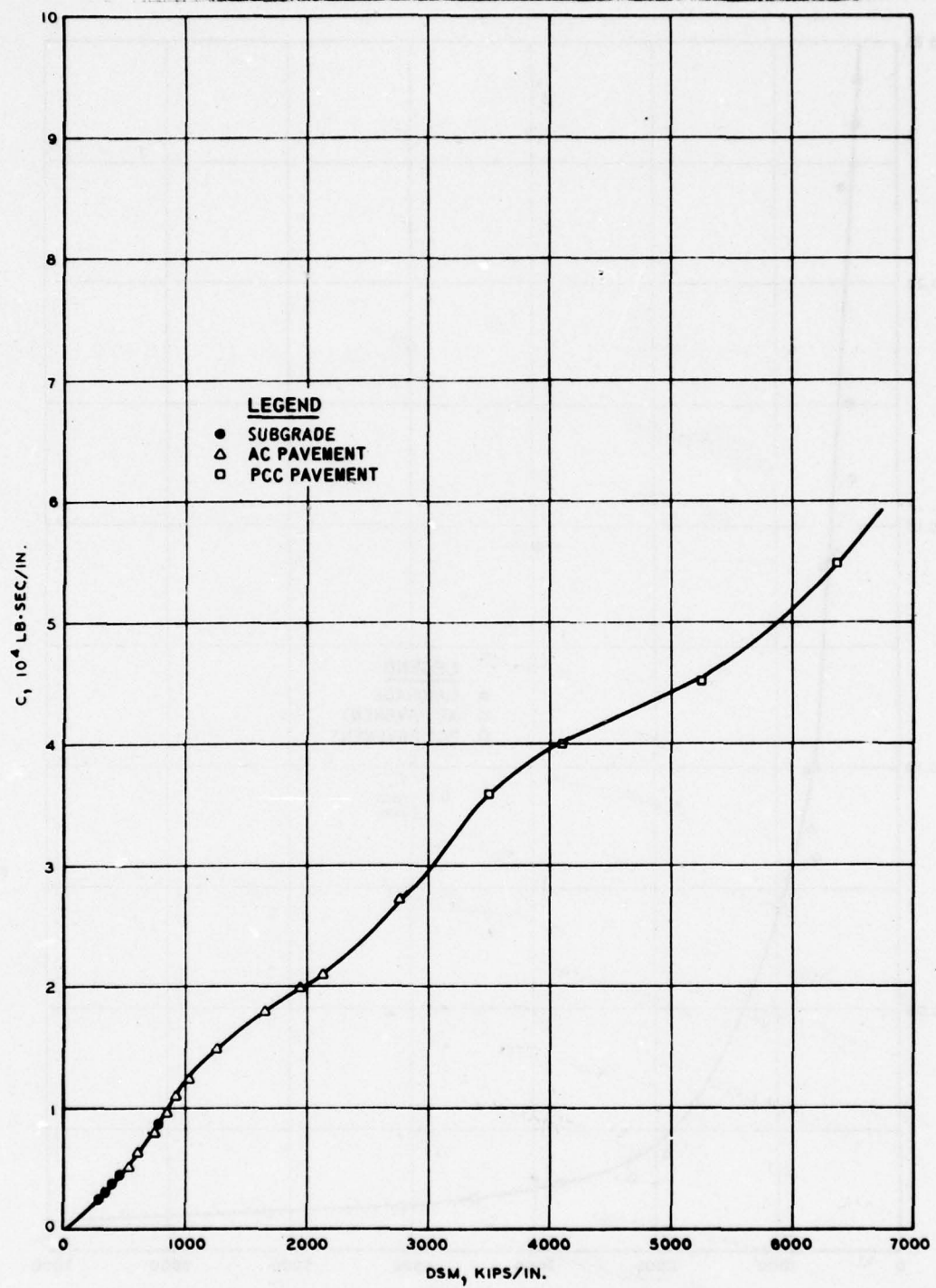


Figure 9. Damping constant versus measured DSM

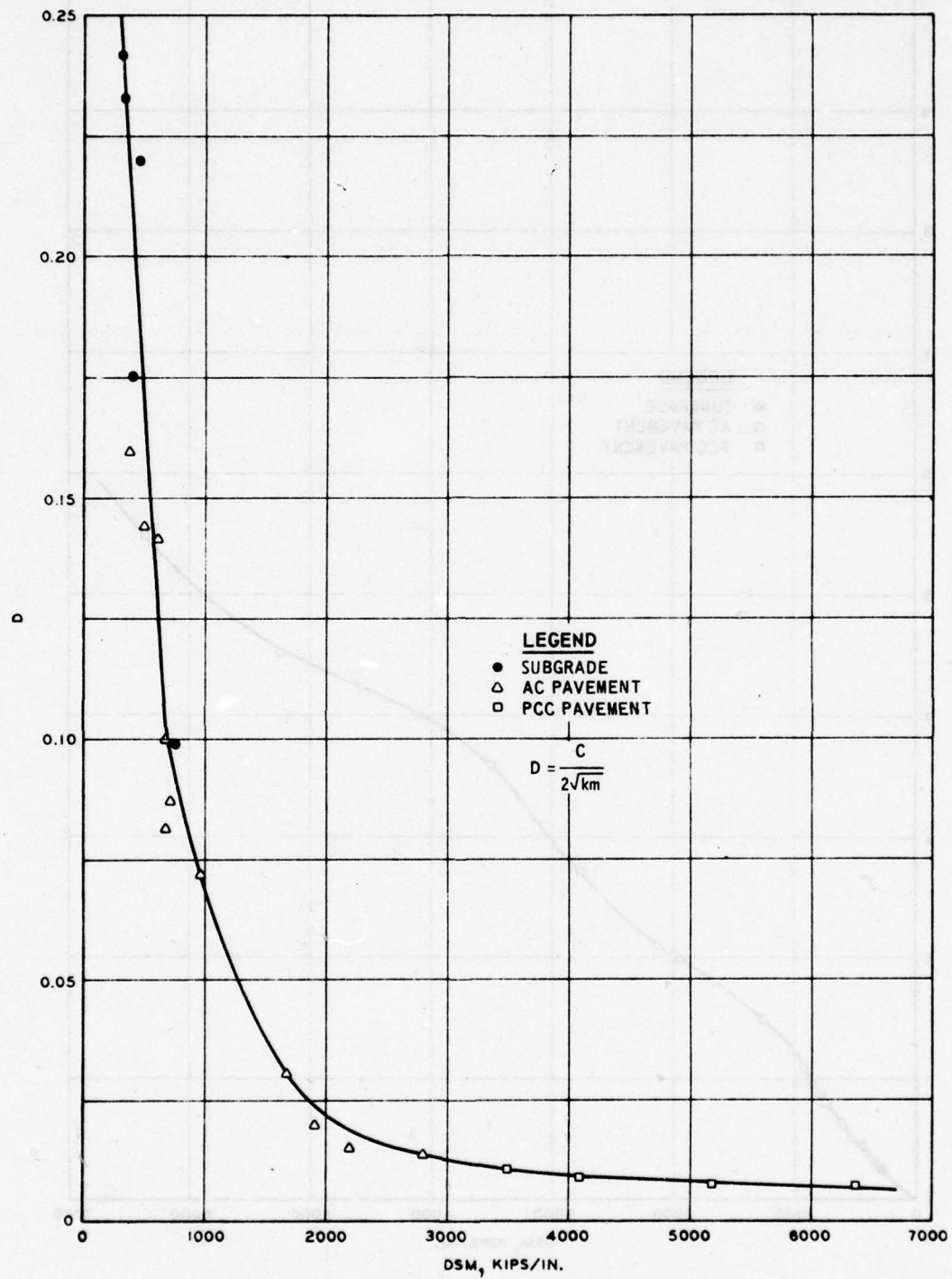
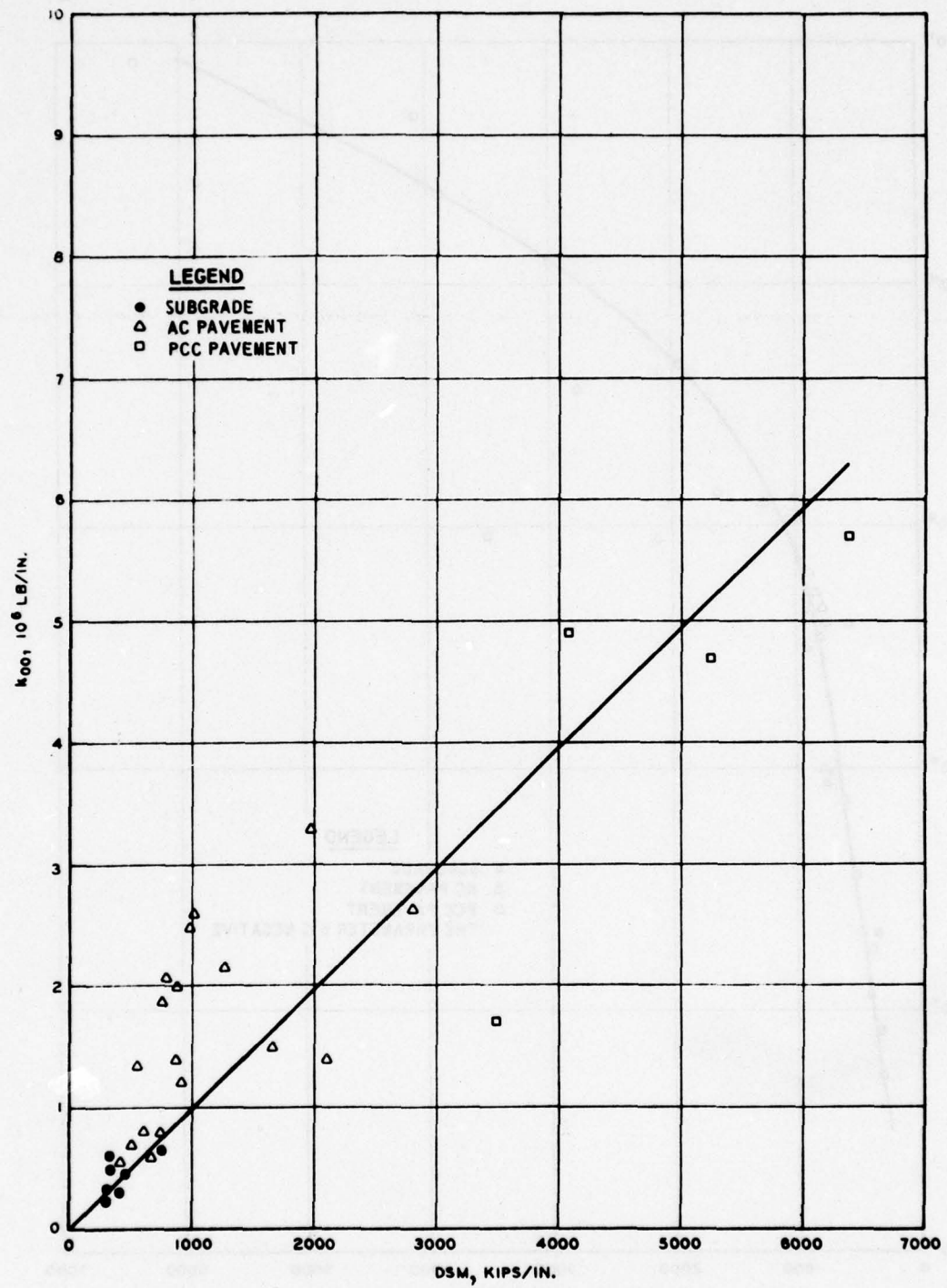


Figure 10. Damping ratio versus measured DSM



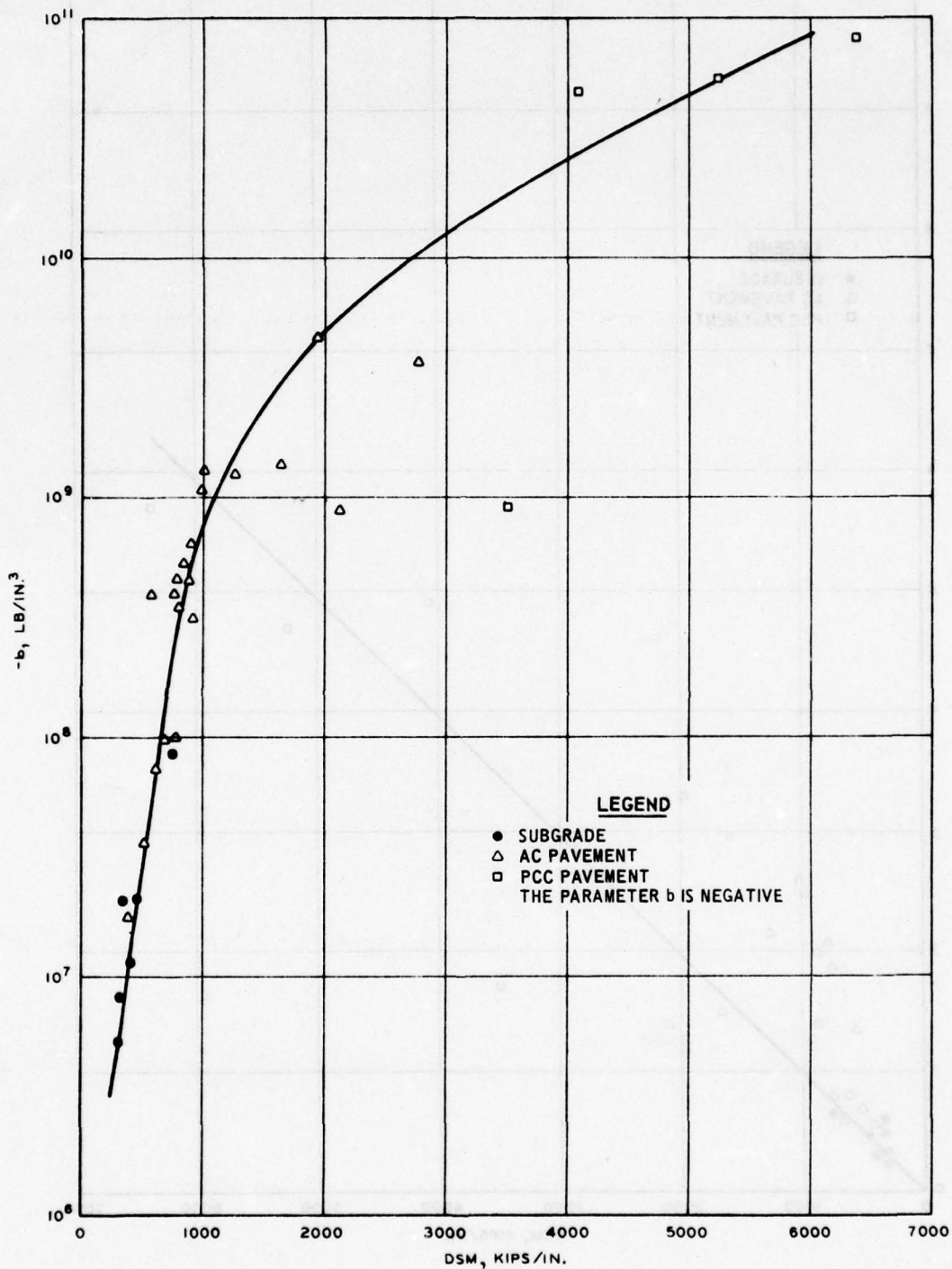


Figure 12. Third-order static nonlinear parameter versus measured DSM

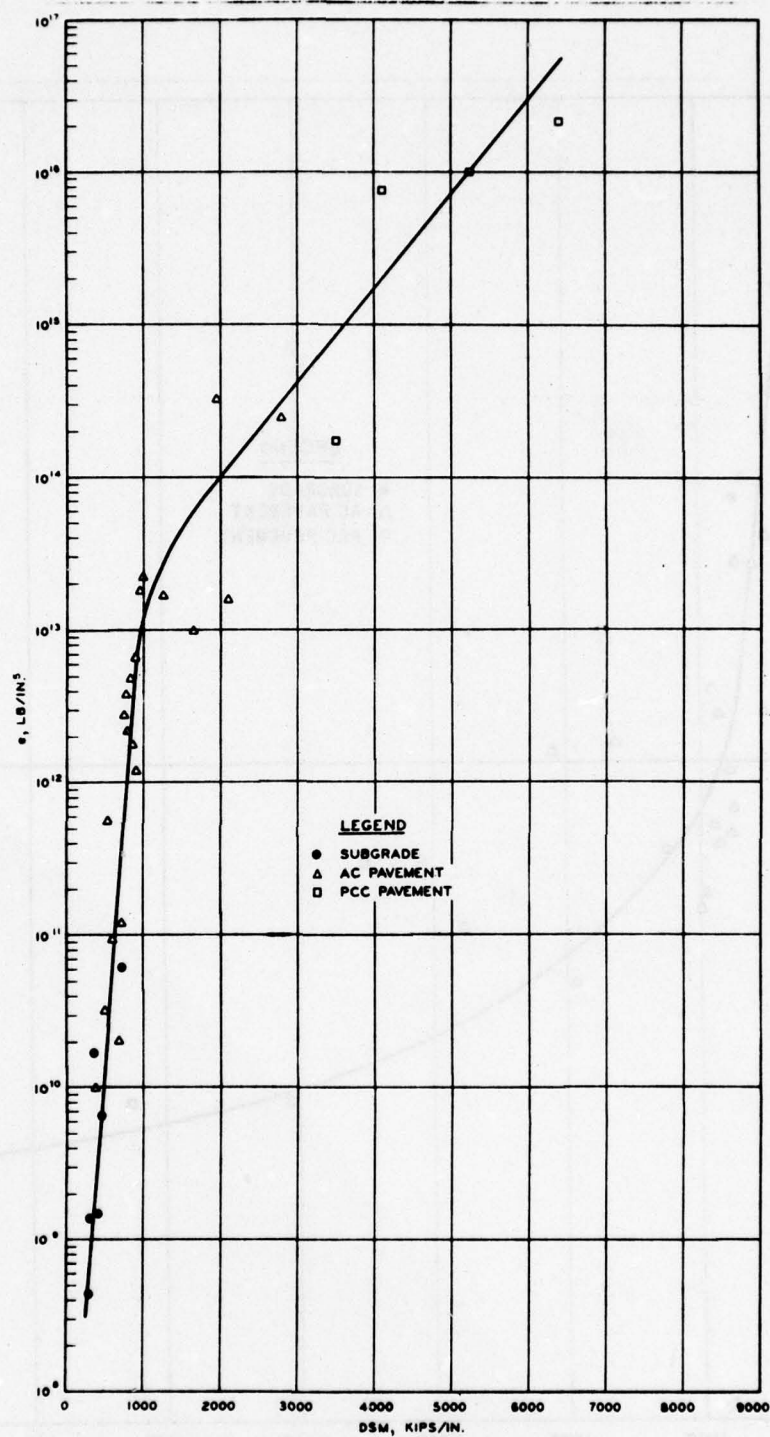


Figure 13. Fifth-order static nonlinear parameter versus measured DSM

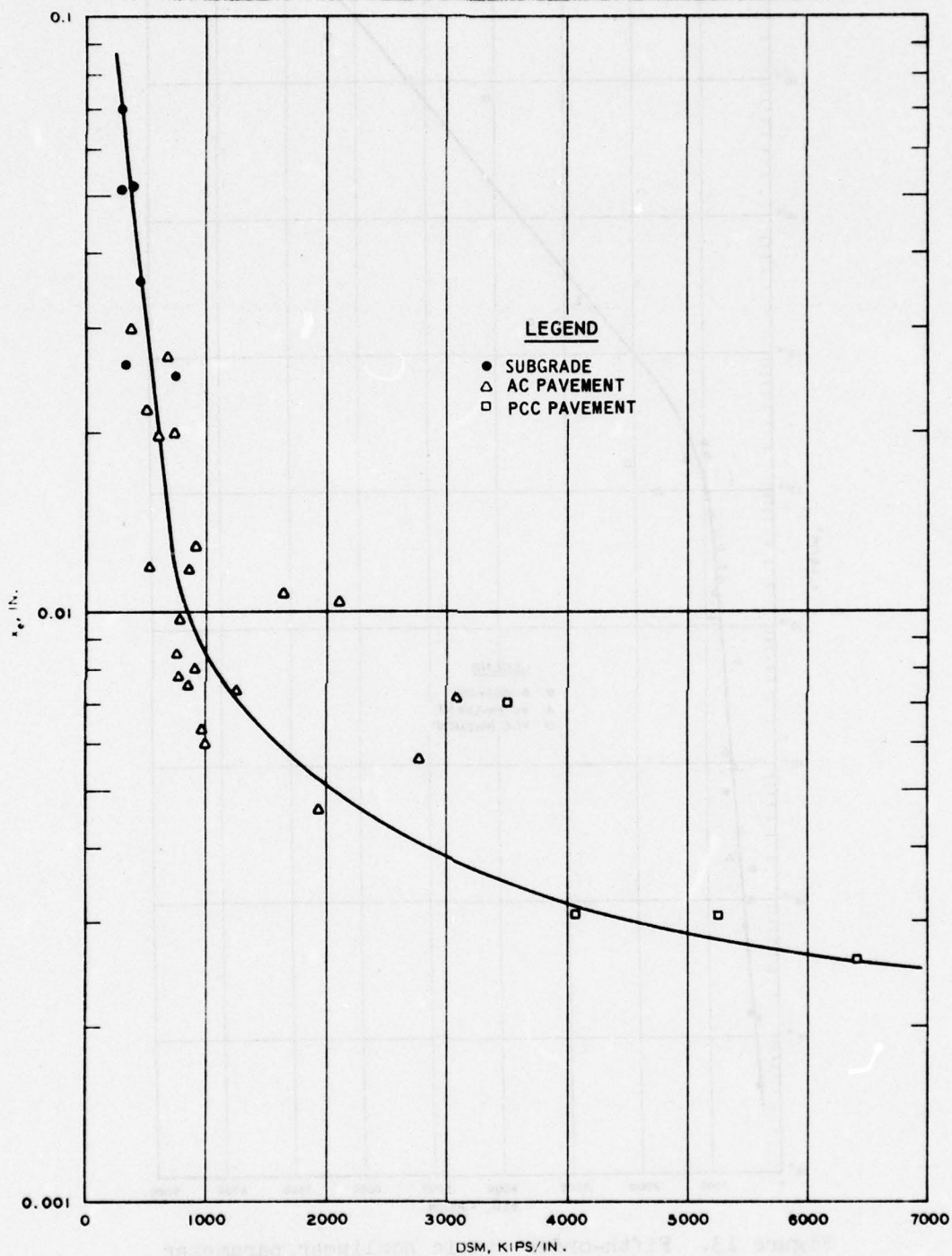


Figure 14. Static elastic displacement versus measured DSM

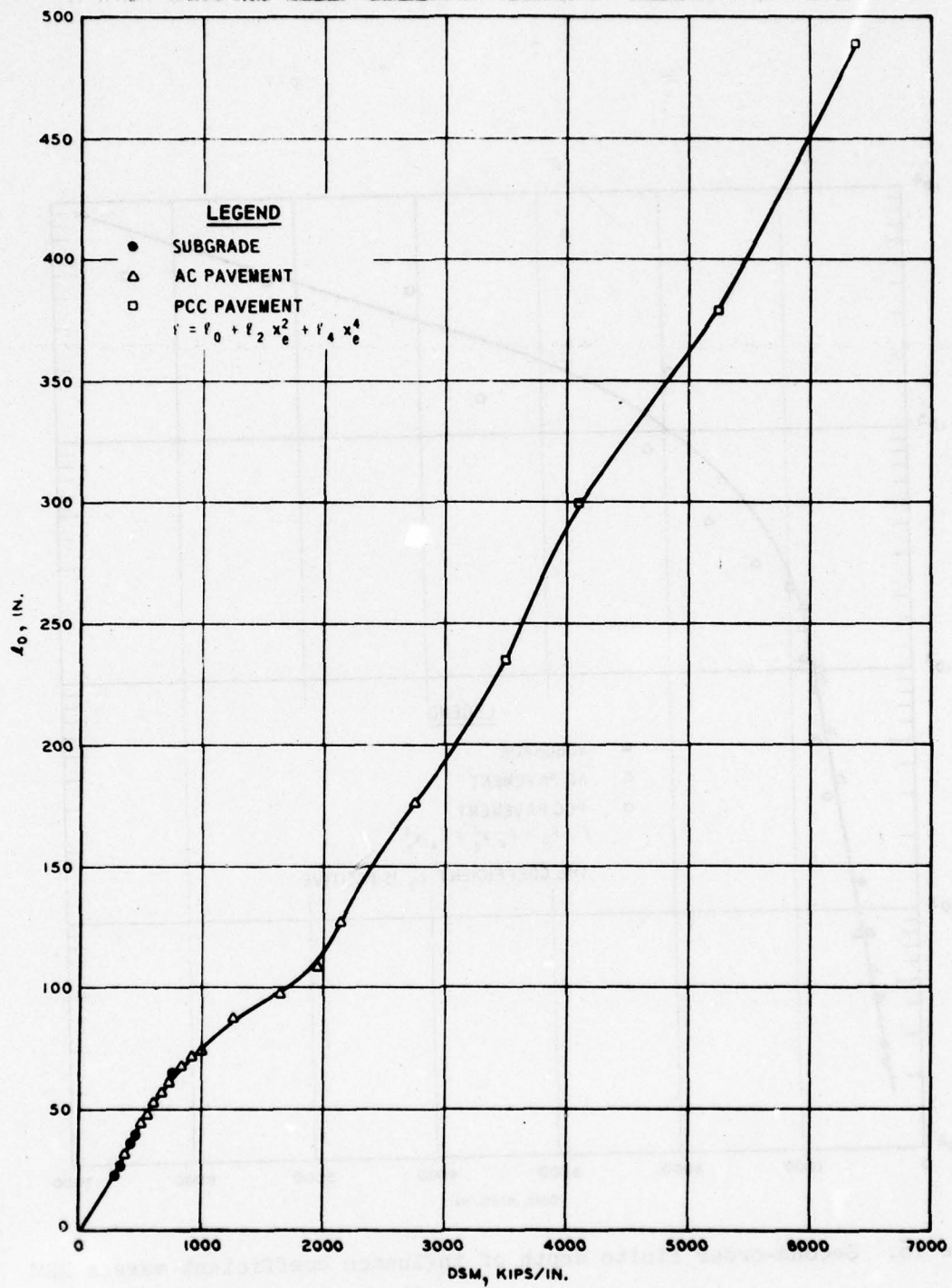


Figure 15. Leading term of the finite depth of influence versus measured DSM

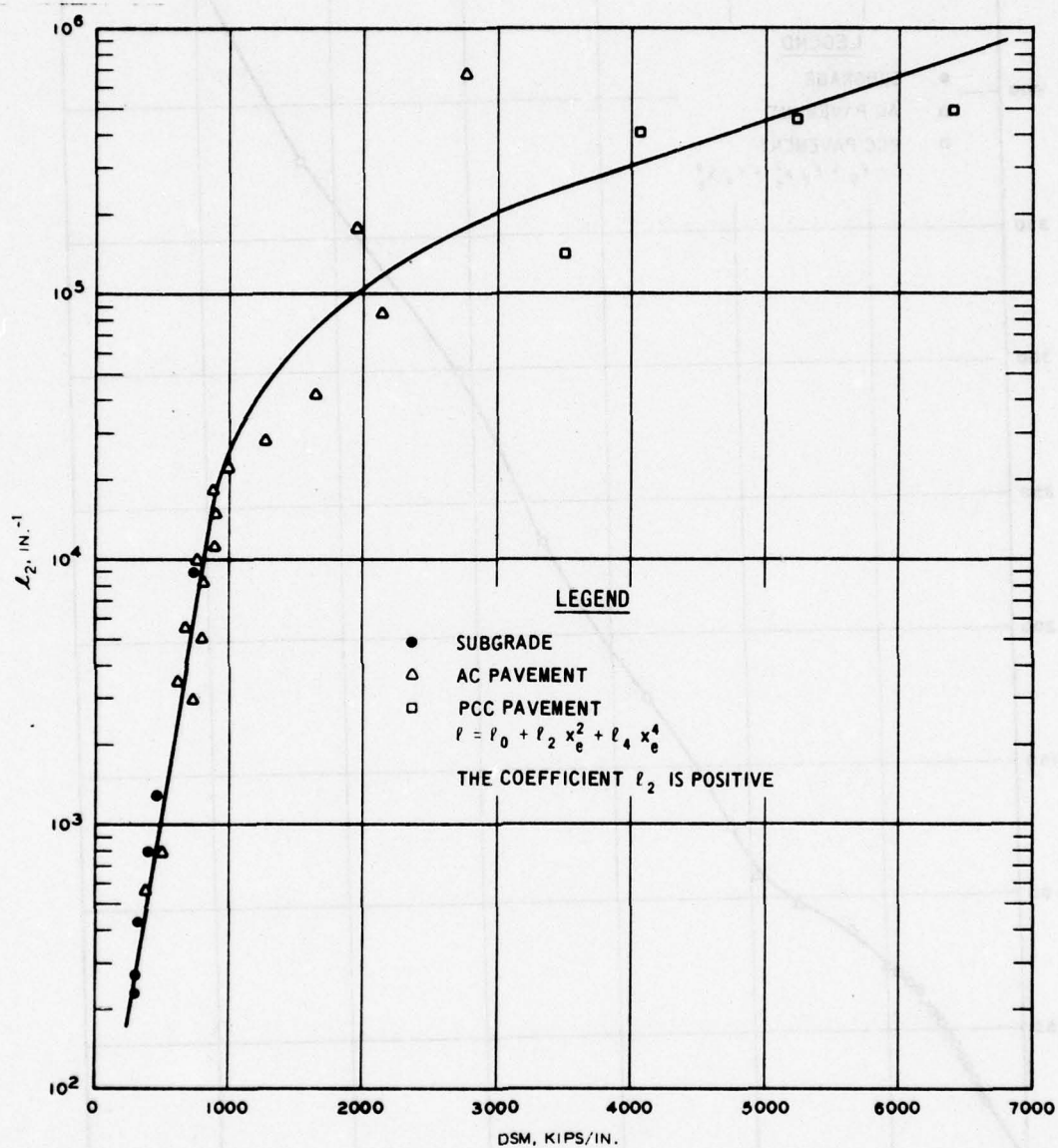


Figure 16. Second-order finite depth of influence coefficient versus DSM

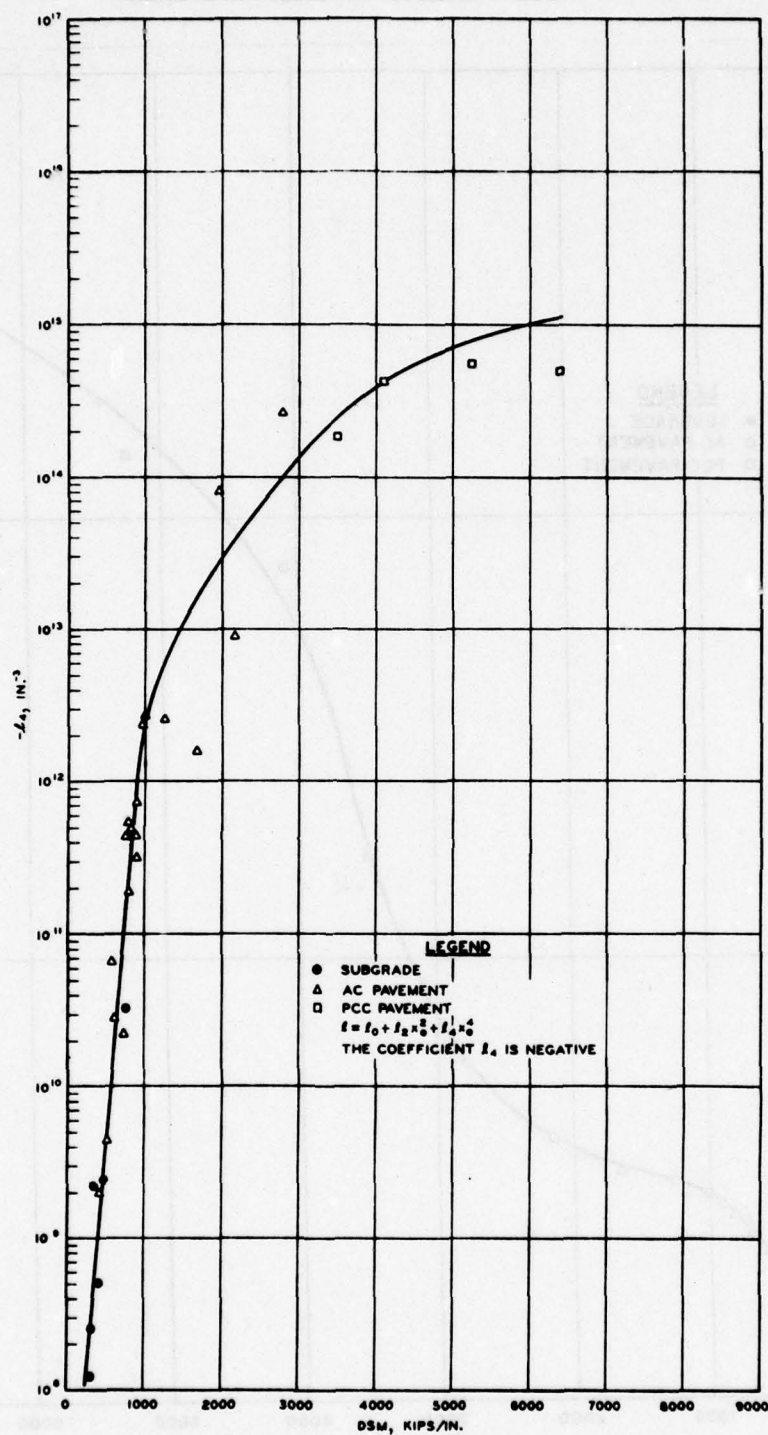


Figure 17. Fourth-order finite depth of influence coefficient versus measured DSM

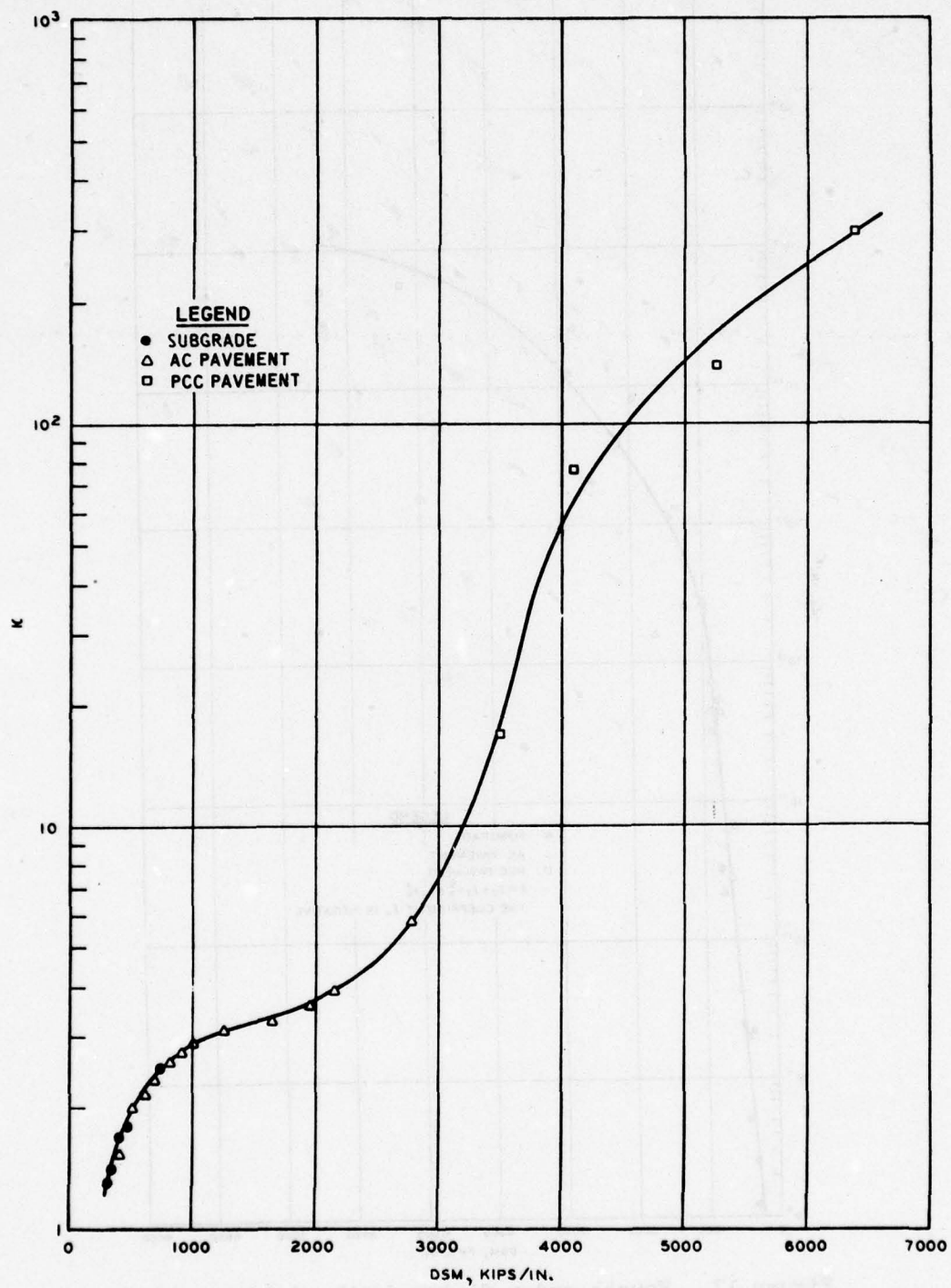


Figure 18. Lateral stress spreading parameter versus measured DSM

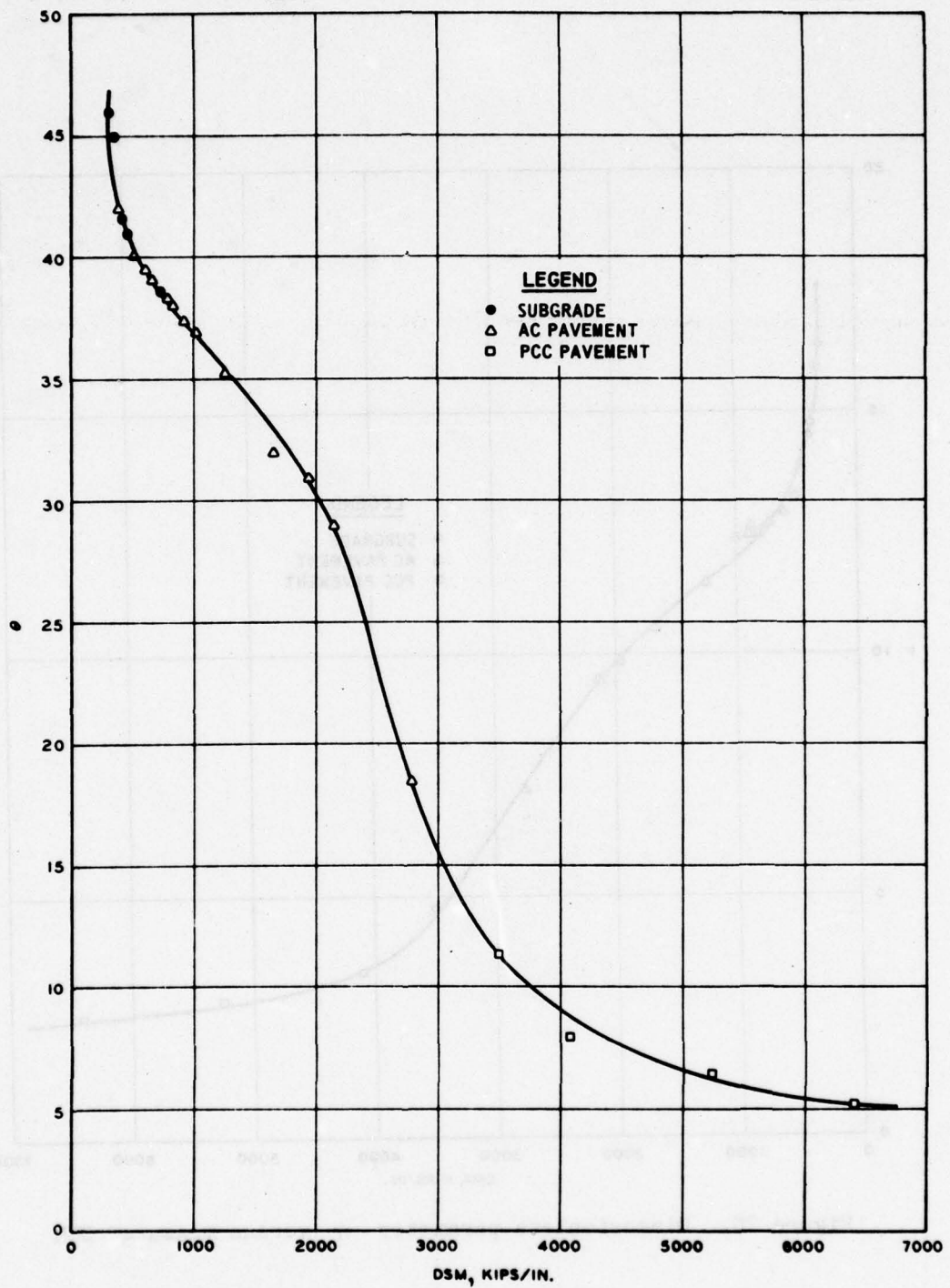


Figure 19. Dimensionless parameter θ versus measured DSM

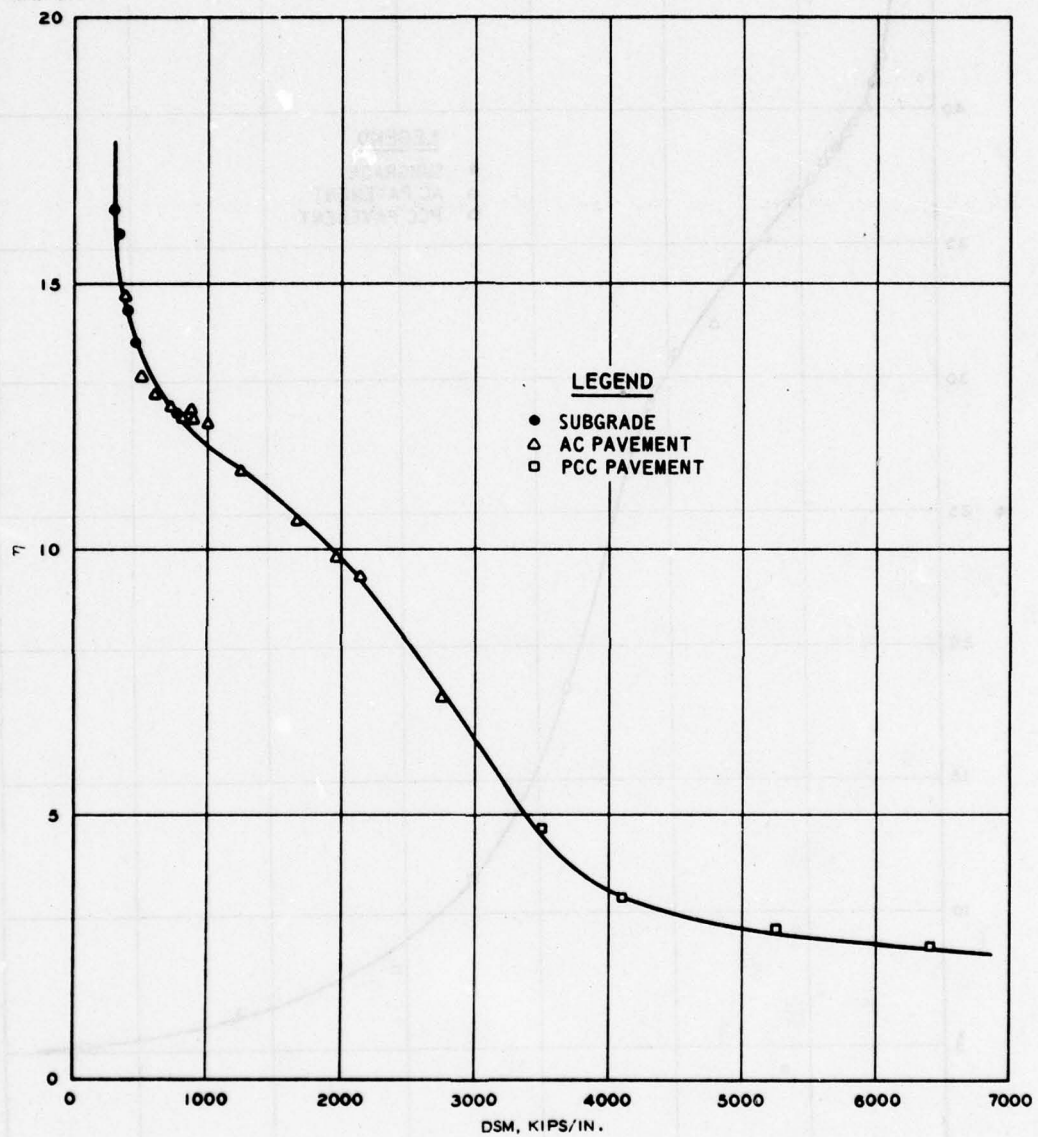


Figure 20. Dimensionless parameter η versus measured DSM

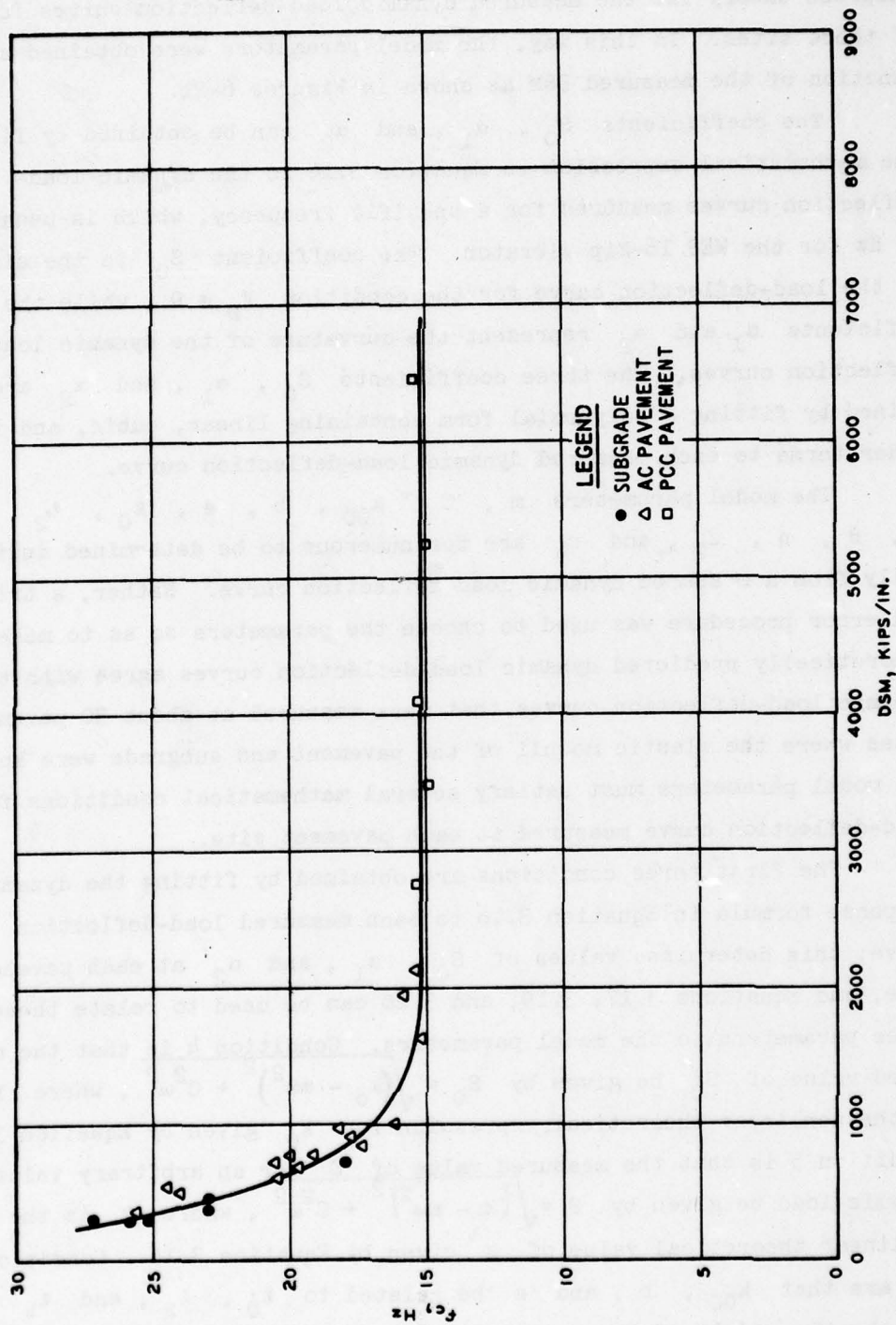


Figure 21. Critical frequency versus measured DSM

obtained for each pavement site made the nonlinear dynamic pavement response theory fit the measured dynamic load-deflection curves for each of these sites. In this way, the model parameters were obtained as a function of the measured DSM as shown in Figures 6-21.

The coefficients S_0 , α_1 , and α_2 can be obtained by fitting the mathematical expression in Equation 3.16 to the dynamic load-deflection curves measured for a specific frequency, which is usually 15 Hz for the WES 16-kip vibrator. The coefficient S_0 is the slope of the load-deflection curve for the condition $F_D = 0$, while the coefficients α_1 and α_2 represent the curvature of the dynamic load-deflection curves. The three coefficients S_0 , α_1 , and α_2 are obtained by fitting a polynomial form containing linear, cubic, and fifth-order terms to each measured dynamic load-deflection curve.

The model parameters m , C , k_{00} , b , e , l_0 , l_2 , l_4 , κ , θ , η , ϵ_2 , and ϵ_4 are too numerous to be determined individually from a measured dynamic load-deflection curve. Rather, a trial and error procedure was used to choose the parameters so as to make the theoretically predicted dynamic load-deflection curves agree with the dynamic load-deflection curves that were measured at about 30 pavement sites where the elastic moduli of the pavement and subgrade were known. The model parameters must satisfy several mathematical conditions for a load-deflection curve measured at each pavement site.

The first three conditions are obtained by fitting the dynamic response formula in Equation 3.16 to each measured load-deflection curve; this determines values of S_0 , α_1 , and α_2 at each pavement site, and Equations 3.17, 3.19, and 3.20 can be used to relate these three parameters to the model parameters. Condition 4 is that the measured value of S_0 be given by $S_0 = \sqrt{(k_0 - m\omega^2)^2 + C^2\omega^2}$, where k_0 is the nonlinear theoretical expression for k_0 given by Equation 3.7. Condition 5 is that the measured value of S for an arbitrary value of dynamic load be given by $S = \sqrt{(k - m\omega^2)^2 + C^2\omega^2}$, where k is the nonlinear theoretical value of k given by Equation 3.15. Conditions 6-8 are that k_{00} , b , and e be related to l_0 , l_2 , and l_4 and the elastic moduli of the pavement layers by Equations 3.88-3.90 of

Reference 4. Condition 9 is that the static elastic displacement of the pavement surface be comparable to the dynamic elastic displacement of the pavement surface. Condition 10 is that the parameter ℓ_4 be negative so that the finite depth of influence Equation 3.26 is convergent. Condition 11 is that the coefficients κ , ℓ_0 , ℓ_2 , and ℓ_4 be increasing functions of the measured DSM value of a pavement. Condition 12 is that the theoretical and measured values of the DSM be equal.

The model parameters were adjusted for each pavement site of known structure and measured DSM, subject to the mathematical conditions mentioned above, until a pattern of variation of these parameters as a function of DSM was obtained. Each data point in Figures 6-21 refers to a pavement site for which the elastic moduli of the subgrade and pavement layers were assumed to be known and for which a measured DSM value has been obtained. These parameters have been derived from dynamic load-deflection curves measured at 15 Hz, and the model parameters are, therefore, associated with a frequency of 15 Hz. There is a possibility that these parameters are intrinsically frequency-dependent. A more detailed account of the behavior and physical meaning of the model parameters will now be given.

3.3.1 EFFECTIVE SPRING CONSTANTS k AND k_0

According to Equation 3.7, the spring constant k_0 is a function of the static elastic displacement of the pavement surface beneath the vibrator baseplate, and according to Equation 3.15, the spring constant k that enters the impedance calculation depends on both the static and dynamic deflections of the pavement surface beneath the vibrator baseplate. As shown in Figures 6 and 7, the spring constants k and k_0 are increasing functions of the measured DSM value of the pavement. The values of the dynamic spring constant k include the nonlinear effects of the static and dynamic loads generated by the vibrator, and therefore the value of k cannot be directly compared with the theoretical values of a spring constant that would be obtained from a static linear layered elastic theory such as is described by the Chevron computer program.

The values of the spring constant k_0 include the nonlinear effects of the static load exerted by the vibrator, so that k_0 also cannot be directly compared with the value of a spring constant that is predicted by a linear layered elastic theory. Because k and k_0 are nonlinear spring constants, they must be comparable in magnitude.

3.3.2 EFFECTIVE MASS m

The effective mass that enters the impedance calculation is shown in Figure 8. The experimental data obtained using the WES 16-kip vibrator on several different types of pavements and subgrades shows that the effective mass which enters the calculation of the dynamic stiffness of a pavement or subgrade surface is not related to the moving mass of the vibrator m_v . The value of the effective mass depends on the structure of the pavement and subgrade and is an increasing function of the DSM value of the pavement or subgrade. For pavements it is much larger than the vibrator mass $m \gg m_v$, but for subgrades it may be equal to or less than the vibrator mass.

The dynamic load generated by the WES 16-kip vibrator is applied to the pavement surface through a moving mass whose weight W_v is 16 kips and whose mass m_v is W_v/g , where g is the acceleration of gravity. For the WES 16-kip vibrator, the vibrator mass is $500 \text{ lb} \times \text{sec}^2/\text{ft}$. An analysis of the measured dynamic load-deflection curves for pavements using a spring model with the elements k , m , and C shows that in general $m \gg m_v$. This means that the lumped effective mass m that occurs in the Kelvin model of the dynamic response of a pavement is associated with the motion of the pavement and subgrade, and is not directly related to the vibrator mass that is used to excite the pavement surface. Therefore the pavement and subgrade system is associated with an effective lumped mass as well as a spring constant and a damping constant. The elastic, inertial, and damping properties of a pavement-subgrade half-space cannot be separated from each other, and any half-space that has the elements k and C associated with it must of necessity also have an effective mass m .

The lumped effective mass of a pavement-subgrade half-space

determines in part the dynamic properties of the half-space. Equation 3.24 shows that the effective mass determines in part the value of the critical frequency associated with the dynamic load-deflection curves of a pavement. As seen by Equations 3.16-3.20, the effective mass plays an important role in determining all of the parameters required to describe the dynamic load-deflection curves.

Because it appears in the expression for the impedance of the pavement surface, the effective mass is a measure of the inertial effects of the pavement and subgrade and has no direct connection with the mass of the vibrator. The concept of effective mass occurs in linear systems as well as nonlinear systems and plays an important role in acoustics.^{10,11} The effective mass reflects mainly the inertial effects associated with the mechanical motion of the material in the pavement and subgrade; it produces a large contribution to the value of the DSM. The large value of the effective mass indicates that the inertial term $m\omega^2$ is comparable to the spring constant term k . The effective mass that is presented in Figure 8 was determined from vibratory non-destructive tests done at 15 Hz. The effective mass may be frequency-dependent, and this dependence would have to be determined from frequency response curve measurements, from the location and size of the dominant resonance frequency peak.

3.3.3 EFFECTIVE DAMPING CONSTANT C AND DAMPING RATIO D

The damping constant C appears in Figure 9 as a function of the DSM value of a pavement or subgrade. The value of C depends on the structure of the pavement and is found to be an increasing function of the measured DSM value of the pavement or subgrade. As shown in Figure 10, the damping ratio $D = C/(2\sqrt{km})$ is a decreasing function of the measured DSM. For the WES 16-kip vibrator operating on subgrades $D \approx 0.3$, while for this vibrator operating on relatively stiff pavements $D \approx 0.01$. Therefore, the damping ratio varies considerably from one site to another and cannot be chosen to be a constant for all pavements.

3.3.4 STATIC ELASTIC PARAMETERS k_{00} , b , AND e

The parameters k_{00} , b , and e are given in Figures 11, 12, and 13, respectively. These coefficients describe the static load-deflection curve which is measured at the surface of the pavement or subgrade and is represented mathematically by Equation 3.12. The coefficient k_{00} determines the linear portion of the static load-deflection curve, while b and e describe the nonlinear portion of this curve. The values of these parameters depend on the structure of the pavement and subgrade. Detailed calculations of the dependence of k_{00} , b , and e on the structure of the pavement and subgrade have been performed.⁴ The value of the parameter b has been found to be negative for all of the pavement and subgrade sites that were investigated, and the parameter e has been found to be positive for all sites. The linear elastic spring constant k_{00} can be compared directly with the spring constant that is predicted by the Chevron layered linear elastic computer program, although it should be remembered that k_{00} is associated with a finite depth of influence of the static stress and strain field, while the Chevron linear elastic calculation has an infinite range of influence. The spring constant k_{00} is related to the elastic moduli of the pavement layers and the subgrade, and does not depend on the magnitude of the static and dynamic loads generated by the vibrator at the pavement surface.

The static elastic parameters k_{00} , b , and e were determined by fitting fifth-order polynomial forms to the measured dynamic load-deflection curves as required by Equation 3.16, and then using Equations 3.7-3.20 to determine these static elastic parameters. There is a considerable amount of scatter in the predicted values of k_{00} , $-b$, and e , but they appear to be generally increasing functions of DSM. As shown in Figure 14, the predicted static elastic displacement under the static load of 16 kips is a decreasing function of the measured DSM.

3.3.5 FINITE DEPTH OF IN- FLUENCE COEFFICIENTS ℓ_0 , ℓ_2 , AND ℓ_4

As mentioned in Section 3.2.4, the nonlinearity of the load-deflection curves measured at a pavement surface can be related to the assumption of a finite depth of influence for the static stress-strain field in the subgrade.⁴ For a linear elastic half-space, the stress-strain field due to a static load acting at the surface extends to an infinite depth and radial distance. A nonlinear load-deflection curve can be derived from the assumption that the static stress-strain field extends to a finite depth and radial distance.⁴

The coefficients ℓ_0 , ℓ_2 , and ℓ_4 are related to the elastic moduli of the pavement layers and to the static elastic parameters k_{00} , b , and e .⁴ The values of the coefficients ℓ_0 , ℓ_2 , and ℓ_4 were obtained by examining a number of dynamic load-deflection curves for pavements of known structure, determining the coefficients k_{00} , b , and e by fitting these load-deflection curves with fifth-order polynomials according to Equation 3.16, and then determining ℓ_0 , ℓ_2 , and ℓ_4 in terms of these coefficients by using Equations 3.74-3.90 of Reference 4, the simplest example of which appears in Equations 3.27-3.30 of the present report.

The finite depth of influence of the static stress-strain field is described by the coefficients ℓ_0 , ℓ_2 , and ℓ_4 that appear in Equation 3.26 and are shown in Figures 15-17. These coefficients depend on the size of the vibrator baseplate and on the structure of the pavement and subgrade.⁴ The coefficients ℓ_0 , ℓ_2 , and ℓ_4 are found to be increasing functions of the measured DSM value of a pavement, and this means that a static load applied to a strong pavement will influence more of the subgrade than would the same load applied to a weak pavement. The static displacement of the pavement surface will be less for the strong pavement than for the weak pavement, but the strain in the subgrade under the strong pavement will extend to a greater depth than it will under the weak pavement. A static load applied to a weak pavement or soil formation will produce a large displacement which is

localized to the area under the load, while the same load applied to a strong pavement will produce a small displacement which extends over a large volume of material. As shown in Figures 15-17 the signs of ℓ_0 , ℓ_2 , and ℓ_4 are as follows: $\ell_0 > 0$, $\ell_2 > 0$, and $\ell_4 < 0$.

3.3.6 STRESS-STRAIN DISTRIBUTION PARAMETER κ

The stress-strain field in the pavement and subgrade beneath the static 16-kip load of the WES vibrator is assumed to be confined to a frustum of a cone whose upper radius is equal to the radius of the vibrator baseplate and whose lower radius is determined by the degree of horizontal spreading of the stress and strain in the pavement and subgrade.⁴ The parameter κ is the ratio of the radius of the lower area of the frustum to that of the upper area. If $\kappa = 1$, the stress and strain would not spread horizontally and would be confined to a vertical cylinder in the pavement and subgrade directly beneath the vibrator baseplate. Values of κ larger than unity indicate that the stress-strain field extends into the horizontal as well as the vertical regions of the pavement and subgrade. The parameter κ is a measure of the lateral spreading of the stress-strain field in a pavement, and indicates how much of the static load applied to the pavement surface is transmitted to the subgrade at a point directly beneath the load.

An important function of a pavement is protection of the subgrade from excessive stress and strain which would cause undesirable plastic flow in the subgrade. The structure of a pavement is designed in part with this purpose in mind. Therefore, it is important to know the dependence of the lateral stress-strain spreading factor κ on the structure (strength) of pavements. For the case of a subgrade alone, the parameter κ is related to the parameters k_{00} , b , e , ℓ_0 , ℓ_2 , and ℓ_4 in the manner indicated in Equations 3.27-3.30. The relationship of these parameters for the case of a pavement over a subgrade is given in detail in Reference 4.

The parameter κ was determined along with the parameters k_{00} , b , e , ℓ_0 , ℓ_2 , and ℓ_4 by fitting a number of dynamic load-deflection curves to the mathematical expression in Equation 3.16.

Figure 18 gives the parameter κ as a function of the measured DSM of pavements. This figure shows that the lateral spreading of the static stress-strain field increases with the measured DSM of the pavement, and that the lateral spreading of the stress and strain is smaller for AC pavements than for the stronger PCC pavements. This is in agreement with the well-known fact that AC pavements tend to transmit the surface load directly into the subgrade, while PCC pavements tend to diffuse the load over an extended area of the subgrade.

3.3.7 DYNAMIC PARAMETERS θ , η , ϵ_2 , AND ϵ_4

When the WES 16-kip vibrator operates on a pavement surface, it produces static and dynamic deflections of the pavement surface. For a hypothetical linear dynamic system, the static displacement of a pavement surface does not enter into the dynamic equation of motion of the pavement surface.⁴ For a nonlinear dynamic pavement-subgrade system, the static displacement of the pavement surface appears in the dynamic equation of motion of the pavement surface, and the static and dynamic deflections are not independent (see Equation 3.3) and must be calculated jointly from the dynamic equations of motion.⁴

The parameters θ , η , ϵ_2 , and ϵ_4 appearing in Equations 3.6-3.11 represent a simple approximate way of treating the interdependence of the static and dynamic displacements of the pavement surface. These parameters describe the approximation of writing Equation 3.4 in the form of Equation 3.5 which brings the equation of motion into the solvable form of Equation 3.6.

The parameters θ , η , ϵ_2 , and ϵ_4 represent the cross product terms between the nonlinear dynamic and static displacement terms that occur in the dynamic equation of motion of the pavement surface beneath the vibrator baseplate. These parameters describe the dependence of the spring constant k on the dynamic displacement of the pavement surface and the dependence of the spring constant k_0 on the static elastic displacement of the pavement surface. The parameters θ and η appear in Figures 19 and 20, while $\epsilon_2 = 4.0$ and $\epsilon_4 = 17.0$ for the WES 16-kip vibrator.

3.3.8 CRITICAL FREQUENCY

f_c

The dynamic load-deflection curves for medium strength and stronger AC pavements measured with the WES 16-kip vibrator at 15 Hz are generally nonlinear, but when measured at 5, 10, 20, or 25 Hz they are even more nonlinear.⁴ The shape of the measured dynamic load-deflection curves depends on the operating frequency of the vibrator as well as on the structure of the pavement and subgrade. Therefore, for practical reasons, the load sweep tests are conducted at 15 Hz. The theoretical reason for the existence of this special operating frequency can be seen from Equations 3.16 and 3.19, which show that the theoretical dynamic load-deflection curves become less curved when the operating frequency of the vibrator is given by Equation 3.24. From Equation 3.24, it is apparent that the critical frequency is a function of pavement structure (strength).

The values of the critical frequency that appear in Figure 21 as a function of DSM were determined from Equation 3.24 where k_0 and m are parameters that were determined by analyzing dynamic load-deflection curves measured at a frequency of 15 Hz. For the pavement sites considered in Figure 21, the critical frequency was not obtained by direct measurement of dynamic load-deflection curves at different frequencies in the neighborhood of f_c to see if indeed these load-deflection curves become less curved at the critical frequency. Such tests would be desirable, but at present these data do not exist. Reference 4 shows some evidence of a straightening effect at a frequency of 15 Hz as compared to dynamic load-deflection curves measured at 5, 10, 20, and 25 Hz. The values of the critical frequency that appear in Figure 21 were obtained indirectly from the parameters k_0 and m that were obtained from load-deflection curves measured at 15 Hz.

As shown in Figure 21, the critical frequency is a slowly decreasing function of the measured DSM value of pavements and subgrades. The critical frequency is a decreasing function of the pavement strength because the effective mass that enters the calculation of the critical frequency is an increasing function of pavement strength. For the WES

16-kip vibrator, the critical frequency is approximately 15 Hz for medium strength and stronger AC pavements. Therefore, the reason that the WES 16-kip vibrator is generally operated at 15 Hz is that this frequency is the critical frequency for most pavements. Figure 21 also shows that subgrades have critical frequencies considerably larger than 15 Hz, but this has not yet been confirmed experimentally by actually doing tests at these higher frequencies.

Because the values of k and k_0 are comparable, it follows that the resonance frequency f_R is approximately equal to the critical frequency f_c , and therefore Figure 21 may be considered to give the resonance frequency as a function of measured DSM. The resonance frequency for most AC and PCC pavements is about 15-20 Hz. Figure 21 shows that the resonance frequency for subgrades is somewhat higher. The peak frequency f_M is about 18-20 Hz for subgrades and 15-20 Hz for pavements.

3.3.9 GENERAL DISCUSSION OF MODEL PARAMETERS

The spring constants k and k_0 include the nonlinear effects of the static and dynamic deflections of the pavement surface (Equations 3.7 and 3.15) and are therefore not simply related to the linear elastic spring constant k_{00} . A comparison of Figures 6, 7, and 11 shows that the spring constants k and k_0 have comparable values while the linear elastic spring constant k_{00} is about one fifth of their value. Therefore, the spring constant k that would be obtained directly from measured frequency response data using a linear spring model (k, m, C) cannot be used as a linear elastic spring constant from which to extract the elastic moduli of a pavement and subgrade.

It is the linear elastic spring constant k_{00} that is related to the elastic moduli of the pavement layers and subgrade. The difference in the values of k and k_{00} explains the difficulty that arose in the determination of the subgrade modulus using the frequency response method (Section 2.4.1). In that procedure, the value of k determined from a frequency response spectrum was used directly in the

Chevron layered linear elastic computer program with the result that the predicted value of the Young's modulus of the subgrade was several times larger than the value given by the Shell equation, $E_s = 1500 \text{ CBR}$.

In order to use the frequency response spectrum technique for the determination of E_s , it would be necessary to extract the value of the linear elastic spring constant k_{00} from the value of k that is determined directly from a frequency response spectrum. To do this would require frequency response spectra measured for a series of fixed dynamic loads. Because pavements and subgrades respond nonlinearly to dynamic loads, it is always necessary to have test data for a series of dynamic loads in order to separate the linear elastic spring constant k_{00} from the dynamic spring constant k .

3.4 DETERMINATION OF SUBGRADE YOUNG'S MODULUS FROM DYNAMIC LOAD-DEFLECTION CURVES

The nonlinear dynamic response model that has been outlined in the preceding sections can be used in conjunction with a dynamic load-deflection curve measured at the pavement surface to determine the Young's modulus of the subgrade beneath the pavement. The Young's modulus of the subgrade will be determined by comparing the theoretically predicted dynamic load-deflection curve with the dynamic load-deflection curve that is measured at the pavement surface and finding the value of the subgrade Young's modulus which makes the theoretical pavement response agree with the measured response. A computer program (SUBE described in Appendix B) has been developed which calculates the theoretical dynamic response of a pavement in terms of the elastic moduli of the pavement layers and subgrade and in terms of the empirically determined parameters k_{00} , b , e , κ , l_0 , l_2 , l_4 , θ , η , ϵ_2 , ϵ_4 , m , and C which have been expressed in terms of the measured DSM values of the pavement (Section 3.3). A typical example of the vibratory nondestructive input data to the computer program SUBE is shown in Table 2. The computer program SUBE predicts the value of the subgrade Young's modulus from the measured dynamic load-deflection curves.

In addition to the measured dynamic load-deflection curve (and

Table 2
Input to WES Nonlinear Dynamic Program SUBE;
Site B2A, DSM = 700 Kips/In.

<u>F_D</u> <u>kips</u>	<u>ξ</u> <u>in.</u>
0	0.0
2	0.003
4	0.007
6	0.011
8	0.015
10	0.020
12	0.025
14	0.030

the DSM), the computer program SUBE which predicts the subgrade Young's modulus, requires the elastic moduli of the wearing surface, base, and subbase of the pavement. In this way, only the subgrade Young's modulus is unknown, and this can be determined by requiring agreement between the measured and theoretically predicted load-deflection curves. There are logical methods for estimating the elastic modulus of the wearing surface, base course, and subbase course of a pavement.⁷ In this report, the Young's modulus of the AC wearing surface has been estimated using the Asphalt Institute temperature dependence curve that is given in Figure 22. (Reference 7 gives an extensive literature review of the subject of the temperature dependence of Young's modulus for AC pavements.) The values of the Young's modulus for AC and bituminous base materials are taken from Figure 22 corresponding to the pavement temperature value that existed during the measurement of the dynamic load-deflection curves. The Young's modulus for PCC was taken to be 4×10^6 psi.

The Young's modulus of the base and subbase materials was selected on the basis of measured CBR values using the Shell formula $E_s = 1500 \text{ CBR}$. Reference 7 discusses the validity of the Shell formula

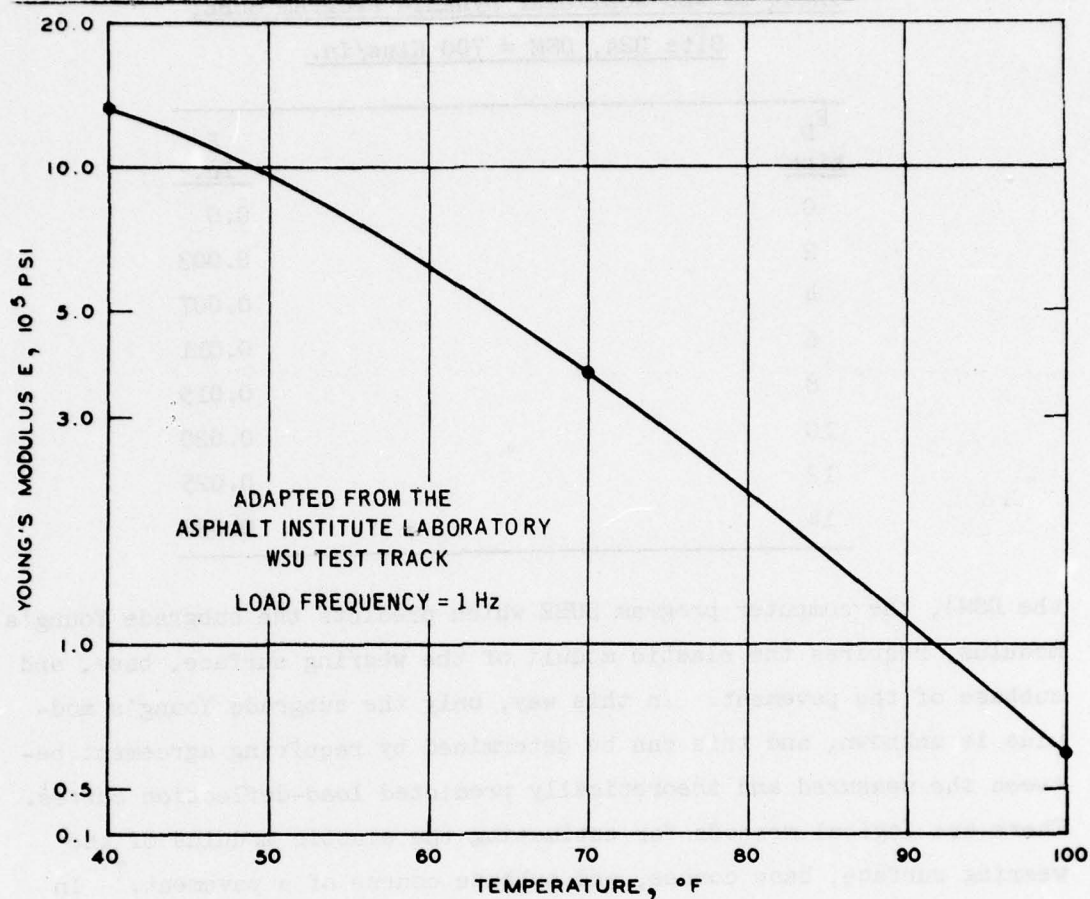


Figure 22. Dependence of Young's modulus of AC on temperature

and describes the wave propagation method on which this relationship is based. The Young's modulus is a linear elastic modulus of a material; i.e. an elastic modulus that is measured under very small dynamic stress and strain levels. The wave propagation method utilizes small amplitude elastic waves which are associated with very small stress and strain values, so that the propagation speed of these waves is a measure of the linear elastic moduli of the pavement material. Therefore, the Shell formula is a reasonable way of selecting the values of the Young's modulus for these pavement materials. The relationship $E_s = 1500 \text{ CBR}$ is a best straight-line fit through a set of data points that have considerable scatter.^{7,8} Therefore, the Shell relationship is at best an

approximation, and considerable uncertainty in the value of the predicted subgrade Young's modulus must be accepted. Table 3 lists the values of the Young's modulus and Poisson's ratio that were used in this report for base course and subbase course materials.

Table 3

Young's Modulus and Poisson's Ratio of
Base and Subbase Materials

<u>Material</u>	<u>Description</u>	<u>Assigned Value of Young's Modulus 10^3 psi</u>	<u>Assigned Value of Poisson's Ratio</u>
Crushed limestone	Crushed limestone	80	0.35
GW	Well-graded gravel	60	0.35
GW-GM	GW and silty gravel	50	0.35
GP	Poorly graded gravel	40	0.35
Stone	Crushed stone	35	0.35
GP-GC	GP and clayey gravels	35	0.35
SP-SM	Poorly graded sand and silty sand	30	0.35
Bituminous concrete	Mineral aggregate and bituminous material	Temperature dependent	0.30

The values of Poisson's ratio were assigned according to the rule: $\nu = 0.2$ for PCC, $\nu = 0.3$ for AC and AC base materials at all temperatures, $\nu = 0.35$ for all other base and subbase materials, and $\nu = 0.35$ for all subgrade soils. This choice of Poisson's ratio produced the most consistent results for the 30 pavement sites of known structure that were examined. The choice of $\nu = 0.35$ for the subgrade soil was found to be necessary in the sense that larger values of Poisson's ratio produced theoretical values of the DSM that were always smaller than the measured values of the DSM, and predicted E_s values could not be obtained in this case. The values of Poisson's ratio

given above are essentially estimates and are not based on laboratory tests done on pavement and subgrade samples taken from the 30 sites that were investigated.

The layer thicknesses that were assigned to the pavement layers were obtained from construction specifications. No measurements of layer thickness were made at the 30 pavement sites that were investigated. Essentially, the entire pavement structure--elastic moduli and layer thicknesses--has been derived by indirect means and not by direct testing.

The value of E_s that is predicted by the WES nonlinear dynamic load-deflection computer program, SUBE, depends on the choice of the values of Young's modulus and Poisson's ratio of each pavement layer and also of the choice of Poisson's ratio of the subgrade. The predicted value of E_s also depends on the values of the pavement layer thicknesses. Errors in the estimation of these pavement parameters will result in errors in the predicted value of the subgrade Young's modulus.

Only a preliminary study has been done on the sensitivity of the predicted values of E_s to the choice of the values of the elastic moduli and thickness of each pavement layer. The results of this study are essentially that the value of the Young's modulus of the wearing surface has more effect on the predicted value of E_s than do the values of the Young's moduli of the base and subbase materials. The predicted value of the subgrade Young's modulus depends strongly on the choice of the value of Poisson's ratio of the subgrade soil. It is imperative that at least the Young's modulus of the wearing surface of AC and PCC pavements be known accurately, and accurate ways of determining this quantity should be developed. It would be of value if a procedure were developed to nondestructively determine the entire pavement structure.

The subgrade Young's modulus can be determined by combining the nonlinear dynamic model with the measured dynamic load-deflection curves. A theoretical description of the dynamic load-deflection curves using the nonlinear dynamic model has been outlined in Section 3.2 and has been discussed in detail in Reference 4. The parameters k_{00} , b , and

e that appear in the nonlinear dynamic model are functions of the Young's modulus of the pavement layers and the subgrade. These Young's moduli appear as parameters in a general expression for the pavement response to a static and dynamic loading and, therefore, are independent of the magnitude of the static and dynamic loading generated by the vibrator. The Young's modulus of the subgrade is a function of the static overburden pressure because the Young's modulus is a function of the static confining pressure. Other types of elastic moduli (such as the resilient modulus) are dependent on the static and dynamic stress conditions in the pavement. However, it will be shown in Section 4 that the nonlinear dynamic theory described in Section 3.2 can also be used to resolve the resilient modulus into a dependence on dynamic and static stress and on a set of parameters which characterize the pavement and subgrade material.

For a choice of Young's modulus and Poisson's ratio of the upper pavement layers, the subgrade modulus is obtained by requiring that the theoretically predicted dynamic load-deflection curves agree with the measured dynamic load-deflection curves. This procedure for determining the subgrade Young's modulus using the computer program SUBE is shown in Figure 23. The pavement and subgrade structures for which the subgrade elastic modulus was predicted are shown in Table 4. The predicted values of the subgrade Young's modulus are presented in Table 4 along with the values of the subgrade Young's modulus that were obtained from the empirical Shell equation $E_s = 1500 \text{ CBR}$. The values of the subgrade Young's modulus predicted by the nonlinear dynamic response theory are in general agreement with those predicted by the empirical equation $E_s = 1500 \text{ CBR}$.

The values of the subgrade Young's modulus obtained by small-amplitude wave propagation tests (Shell method), by the frequency response spectrum method (Section 2), and by the nonlinear dynamic load-deflection curve method (Section 3) must be affected by the overburden pressure on the soil at the top of the subgrade. The Young's modulus of soils (and pavement materials) depends on the magnitude of the confining pressure. The confining pressure on the soil at the top of the

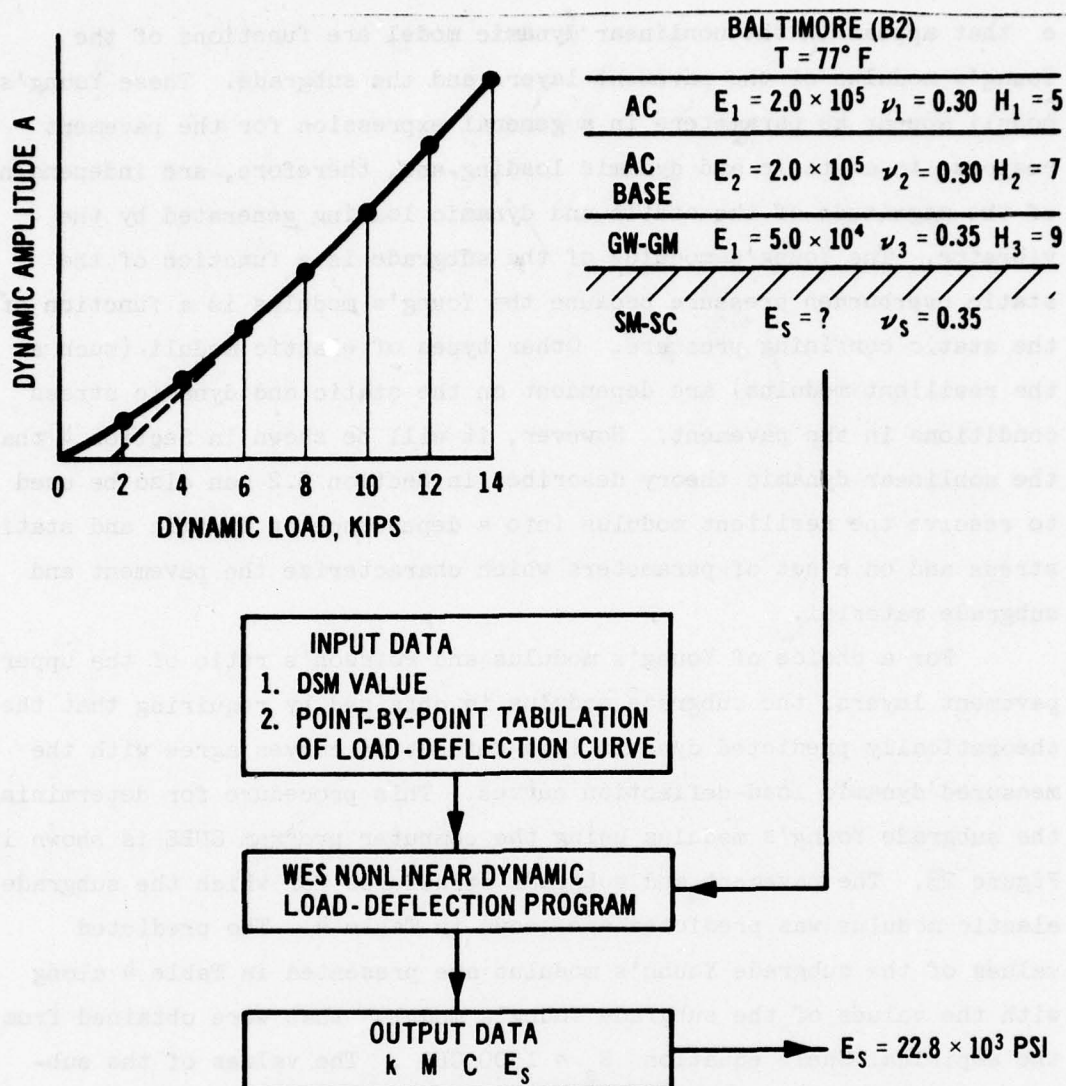


Figure 23. Determination of subgrade modulus from measured dynamic load-deflection curves

subgrade is due to the weight of the pavement material above it, and this overburden pressure will affect the value of the Young's modulus of the soil. The magnitude of the overburden pressure is generally only a few pounds (force) per square inch, but the values of E_s obtained by the Shell method and by the method of nonlinear dynamic load-deflection curves will be affected slightly by this pressure; i.e.,

$$E_s = E_s(\sigma_{OB}) .$$

Table 4

Pavement Structures Investigated

Site	DSM kips/in.	Wearing Surface			Base			Subbase			Subgrade		
		E ₁ psi	v ₁	h ₁ in.	E ₂ psi	v ₂	h ₂ in.	E ₃ psi	v ₃	h ₃ in.	E _s WES Nonlinear psi	v _s	E _s 1500 CBR psi
WES-WEL area subgrade	300	--	--	--	--	--	--	--	--	--	12,300	0.35 Loess	8 12,000
WES hangar No. 4 subgrade	400	--	--	--	--	--	--	--	--	--	10,700	0.35 Lean + heavy clay	31 46,500
TETS-adjacent subgrade	320	--	--	--	--	--	--	--	--	--	27,000	0.35 Lean clay	14 21,000
TETS-poorhouse subgrade	300	--	--	--	--	--	--	--	--	--	15,900	0.35 Lean clay	X X
TETS-adjacent subgrade	450	--	--	--	--	--	--	--	--	--	13,000	0.35 Lean clay	8 12,000
B2A asphaltic concrete	700	230,000 AC T = 77°F	0.3	5	230,000 AC base T = 77°F	0.35	7	32,000 GW-GM	0.35	9	25,000	0.35 SM-SC	14 21,000
N18 asphaltic concrete	770	1,400,000 AC T = 39°F	0.3	3.25	34,000 GP	0.35	6.0	--	--	--	29,600	0.35	18 27,000
WES test area asphaltic concrete	780	100,000 AC T = 90°F	0.3	3.0	80,000 Crushed limestone	0.35	6.0	200,000 Lime-stabilized lean clay	0.35	24.0	6,700	0.35 Heavy clay	4 6,000
WI asphaltic concrete	860	180,000 AC T = 87°F	0.3	9.0	40,000 GW-GM	0.35	5.0	--	--	--	19,500	0.35 SP-SC	20 30,000
Alum creek 123-11 asphaltic concrete	820	1,300,000 AC T = 41°F	0.3	7.8	--	--	--	--	--	--	19,400	0.35 GC	10 15,000

(Continued)

Table 4 (Concluded)

Site	DSM kips/in.	Wearing Surface			Base			Subbase			Subgrade		
		E ₁ psi	v ₁ in.	h ₁ in.	E ₂ psi	v ₂ in.	h ₂ in.	E ₃ psi	v ₃ in.	h ₃ in.	E _s WES Nonlinear psi	v _s CBR	E _s 1500 CBR psi
Alum Creek 123-2 asphaltic concrete	880	1,300,000 AC T = 41°F	0.3	7.8	--	--	--	--	--	--	19,000	0.35 GC	15,000
N23A asphaltic concrete	980	1,300,000 AC T = 41°F	0.3	3.0	1,300,000 AC Base T = 41°F	0.3	3.0	32,000	0.35 GP	7.0	28,000	0.35 SM	27,000
Alum creek 123-23 asphaltic concrete	1000	1,300,000 AC T = 41°F	0.3	7.6	--	--	--	--	--	--	28,000	0.35 GC	25,500
Alum creek 123-32 asphaltic concrete	1230	1,300,000 AC T = 41°F	0.3	7.6	--	--	--	--	--	--	21,000	0.35 GC	25,500
B3 asphaltic concrete	1680	700,000 AC T = 58°F	0.3	5.5	700,000 AC Base T = 58°F	0.3	5.5	35,000	0.35 GW-GM	10.5	30,000	0.35 SP-SM	25,500
W2C asphaltic concrete	1940	300,000 AC T = 73°F	0.3	12.0	60,000 GW-GM	0.35	6.0	--	--	--	35,000	0.35 SP-SC	30,000
Pl4A asphaltic concrete	2120	200,000 AC T = 80°F	0.3	14.0	53,000 GP-GC	0.35	5.0	--	--	--	28,700	0.35 GP-GC	30,000
Pl3 asphaltic concrete	2780	320,000 AC T = 73°F	0.3	15.0	60,000 GP-GC	0.35	5.0	--	--	--	35,000	0.35 GP-GC	30,000
Bl asphaltic concrete	3120	500,000 AC T = 65°F	0.3	5.0	500,000 AC Base T = 65°F	0.3	12.5	28,000	0.35 SP-SM	6.0	21,000	0.35 SM	21,000
WES test area PCC	3500	4,000,000 PCC	0.2	15.0	200,000 AC Base	0.3	6.0	--	--	--	6,800	0.35 Heavy clay	6,000

4. LABORATORY CONFIRMATION OF VIBRATORY NONDESTRUCTIVE FIELD TEST DATA

4.1 GENERAL DESCRIPTION OF THE RESILIENT MODULUS TEST

It is the Young's modulus (evaluated at a static stress level corresponding to the actual stress conditions in the subgrade during the operation of an aircraft) that is entered in the layered elastic computer to determine the allowable static load or pavement overlay thickness for a specified number of repetitions of an aircraft. The nonlinear theory of pavement vibrations that is developed in Section 3 determines the subgrade Young's modulus from nonlinear dynamic load-deflection curves measured at a pavement surface. It is important to have an independent laboratory confirmation of the value of the subgrade Young's modulus that is determined in the field by vibratory nondestructive testing. This section develops a method of correlating the field values of the subgrade Young's modulus with the values of Young's modulus that are extracted from laboratory resilient modulus tests.

When a heavy aircraft operates on a pavement, the stress and strain in the subgrade is sufficiently large to cause nonlinear elastic deformation of the soil. This is expected because even the relatively small load produced by the WES 16-kip vibrator results in nonlinear dynamic load-deflection curves. The resilient modulus is a physical quantity measured in the laboratory which exhibits the nonlinear behaviour of base, subbase, and subgrade materials subjected to repetitive dynamic loads for a series of fixed confining pressures. Because the resilient modulus test exhibits the nonlinear behaviour of soils under dynamic loading, it is a suitable laboratory test to use as a check on the values of the subgrade Young's modulus that are derived from the nonlinear dynamic load-deflection curves measured in the field.

The important physical quantity that must be extracted from laboratory resilient modulus test data is the static elastic Young's modulus. This section applies the nonlinear dynamic theory of pavement response that was outlined in Section 3.2 to the laboratory measurement

of the resilient modulus. The dependence of the resilient modulus on static confining pressure, dynamic deviator stress, and material property parameters will be determined. The material property parameters are independent of the dynamic and static stress and are analogous to the model parameters derived from the dynamic load-deflection curves obtained from the vibratory nondestructive method of testing pavements. The value of Young's modulus can be expressed in terms of the static confining pressure and the material property parameters.

In its natural state, an element of soil in the subgrade is subjected only to the overburden pressure. When a vibrator is operated on the surface of a pavement or subgrade, an additional static and dynamic stress is applied to an element of soil in the subgrade. For the WES 16-kip vibrator, the static load applied to the surface is 16 kips, while the dynamic load can be varied up to 15 kips and is applied sinusoidally with a frequency of 15 Hz. The nonlinear load-deflection curves measured in the field are obtained by sweeping through a range of dynamic loads. The magnitude of the stress and strain field in the pavement layers and subgrade varies with the depth and radial distance from the source and can be calculated by standard elasticity theory with the assumption of linear elasticity.

An appropriate laboratory test will also involve a sweep through a series of dynamic loads for a fixed frequency. The resilient modulus test determines the resilient modulus M_r for a series of dynamic loadings at a fixed frequency and fixed confining pressure.⁷ The laboratory sample for resilient modulus testing is a cylinder with a typical diameter of 3 in. and a length of 6 in. The cylindrical sample is subjected to a static confining pressure, and then a dynamic load is applied in the axial direction. The stress is uniform along the axis of the laboratory sample. The total stress along the axis of the laboratory sample is written as

$$\sigma = \sigma_D + \sigma_S \quad (4.1)$$

where

σ_D = dynamic stress in axial direction of sample

σ_S = confining pressure

The axial dynamic stress is also called the dynamic deviator stress and is written as $\sigma_D = \sigma - \sigma_S$, where σ equals total stress along the axis of the specimen. The resilient modulus has been measured for a number of soil and pavement materials, and M_r has been found to depend on σ_S and σ_D . The results of typical resilient modulus tests appear in Figures 24-27.

The dynamic stress acting along the axial direction of the soil specimen during the laboratory resilient modulus test is applied as a series of pulses in the form of haversines with a pulse of 0.2-sec duration being applied every 3 sec. The characteristic frequency of the dynamic loading on the sample will therefore be in the range of 5 Hz, which is somewhat lower than the frequency of 15 Hz at which the vibratory nondestructive field tests are conducted with the WES 16-kip vibrator. The 0.2-sec duration of the dynamic pulse that is spaced every 3 sec is selected for a standard resilient modulus test done at WES to simulate a moving wheel of an aircraft traveling at 20 to 30 mph. It is possible to alter the equipment to attain a loading frequency comparable to 15 Hz, but this was not done for the resilient modulus tests described here. Future resilient modulus testing should be done at a frequency of 15 Hz when the specific purpose of these tests is a comparison with the results of the vibratory nondestructive tests done with the 16-kip vibrator operating at 15 Hz. Nevertheless, the difference in the frequencies used for these two types of tests requires that an adequate account of frequency effects be included in the theoretical analysis of both laboratory and field vibratory tests.

References 7, 12, and 13 give a description of the resilient modulus and the experimental data which give the dependence of the resilient modulus on static and dynamic stress. At present, there is no theoretical description of the resilient modulus which expresses this quantity in terms of material parameters and in terms of the dynamic and static stress conditions in the pavement and subgrade. In this report, it is assumed that the basic nonlinearity of the dynamic

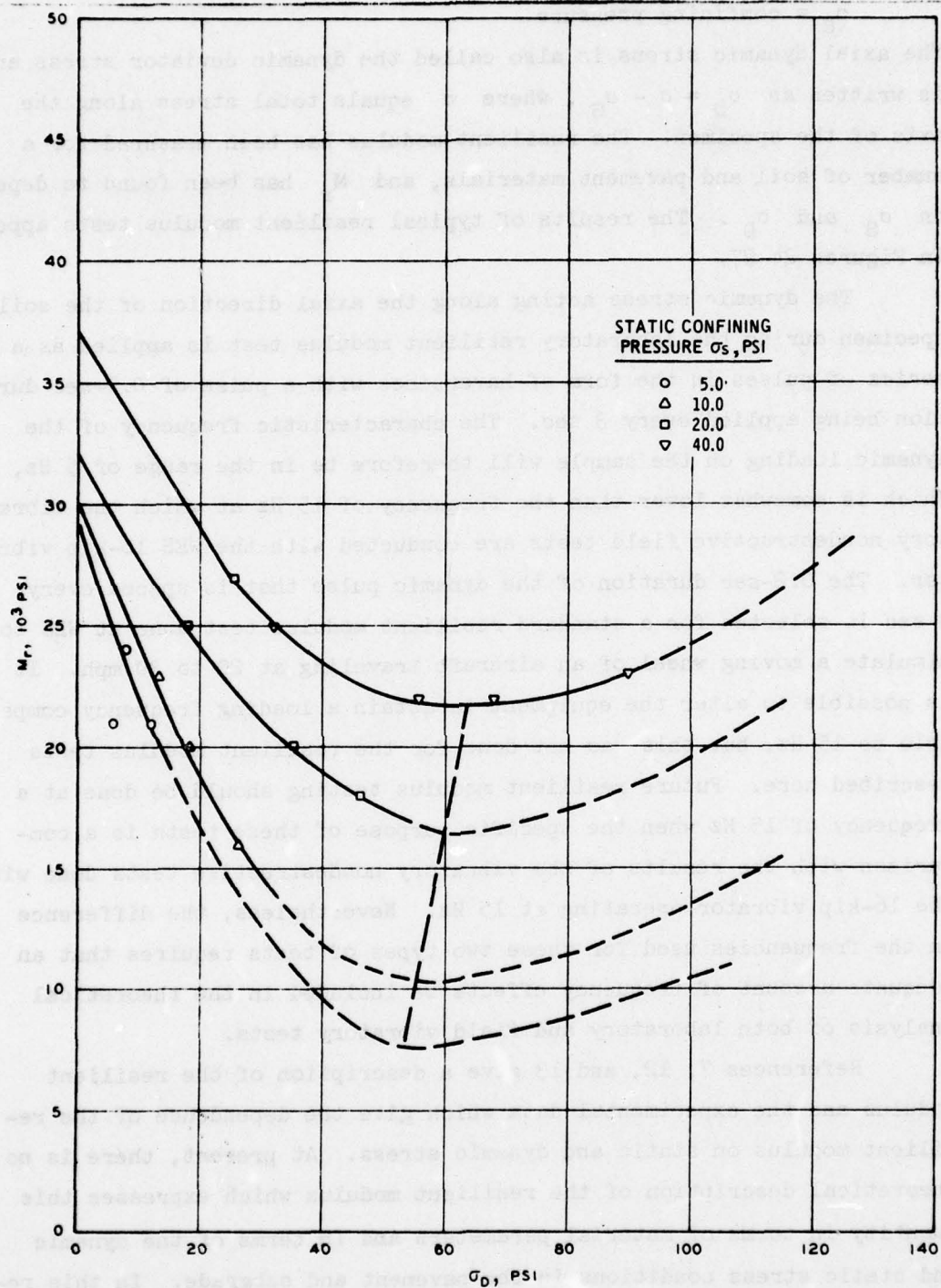


Figure 24. Resilient modulus test results on loess; $\gamma_d = 91.4$ pcf, $w = 20.2$ percent

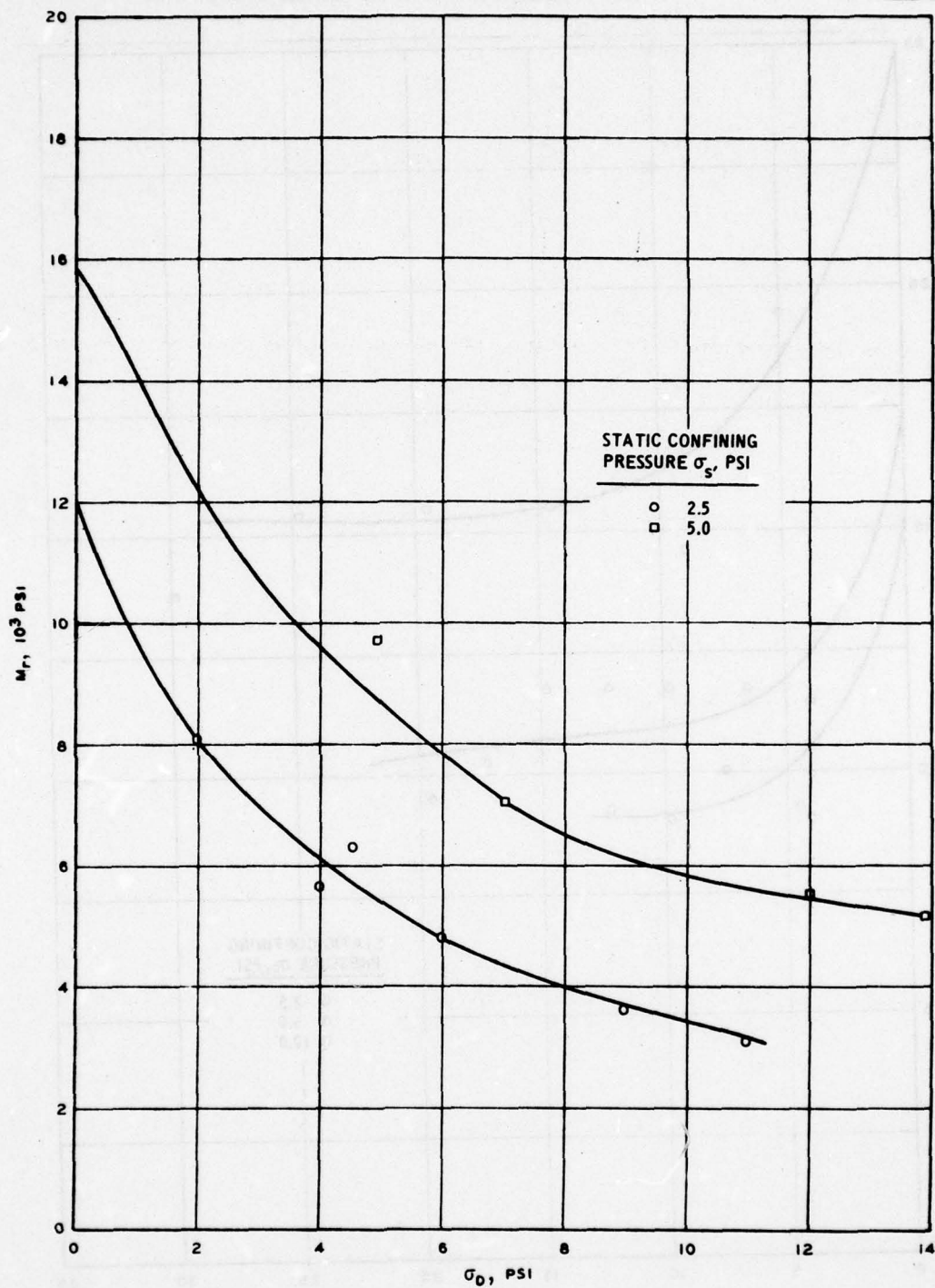


Figure 25. Resilient modulus test results on Poorhouse site specimen No. 10; $\gamma_d = 94.2$ pcf, $w = 24.4$ percent

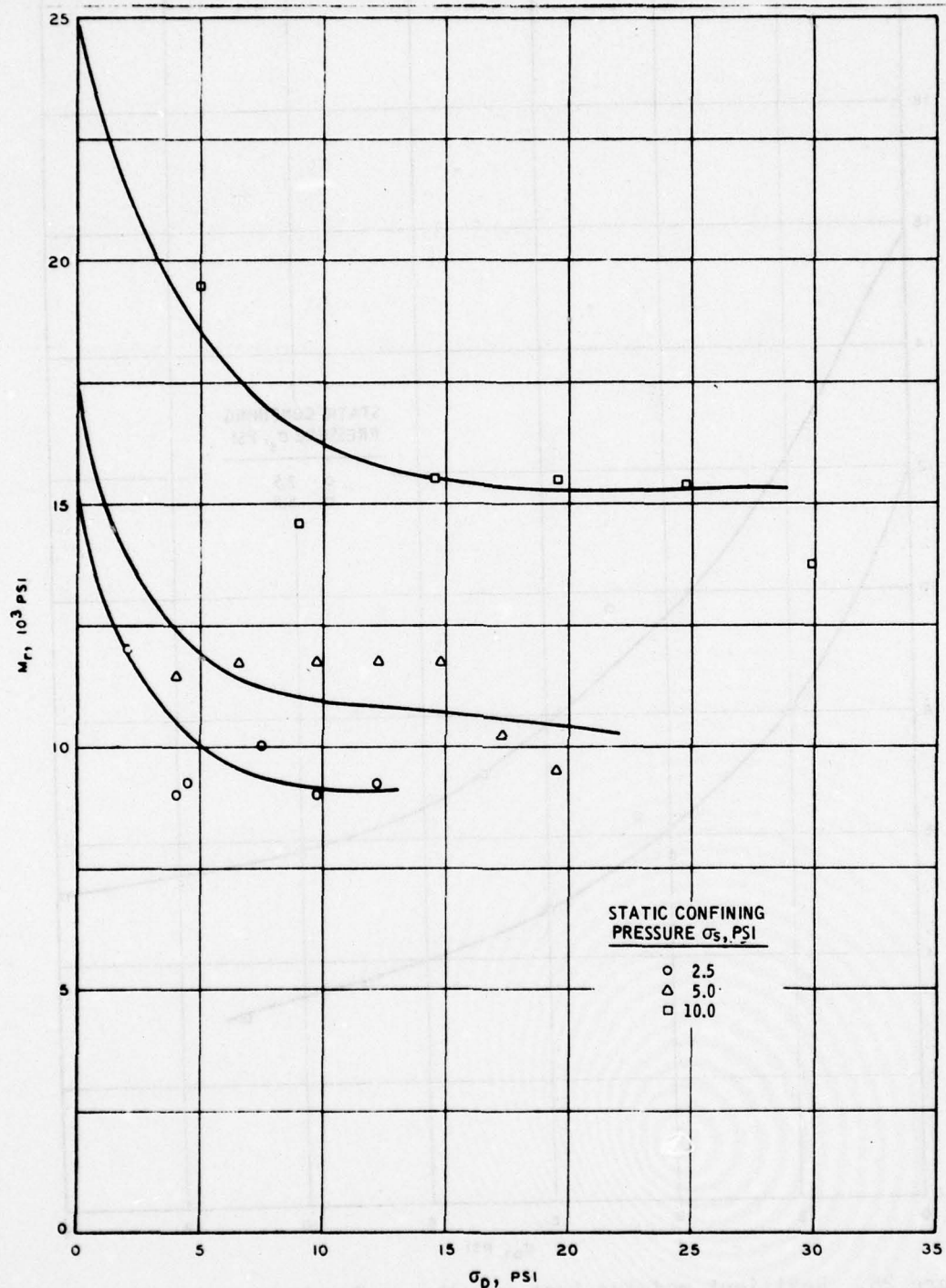


Figure 26. Resilient modulus test results on WES-WEL area specimen No. 12; $\gamma_d = 91.4$, $w = 20.2$ percent

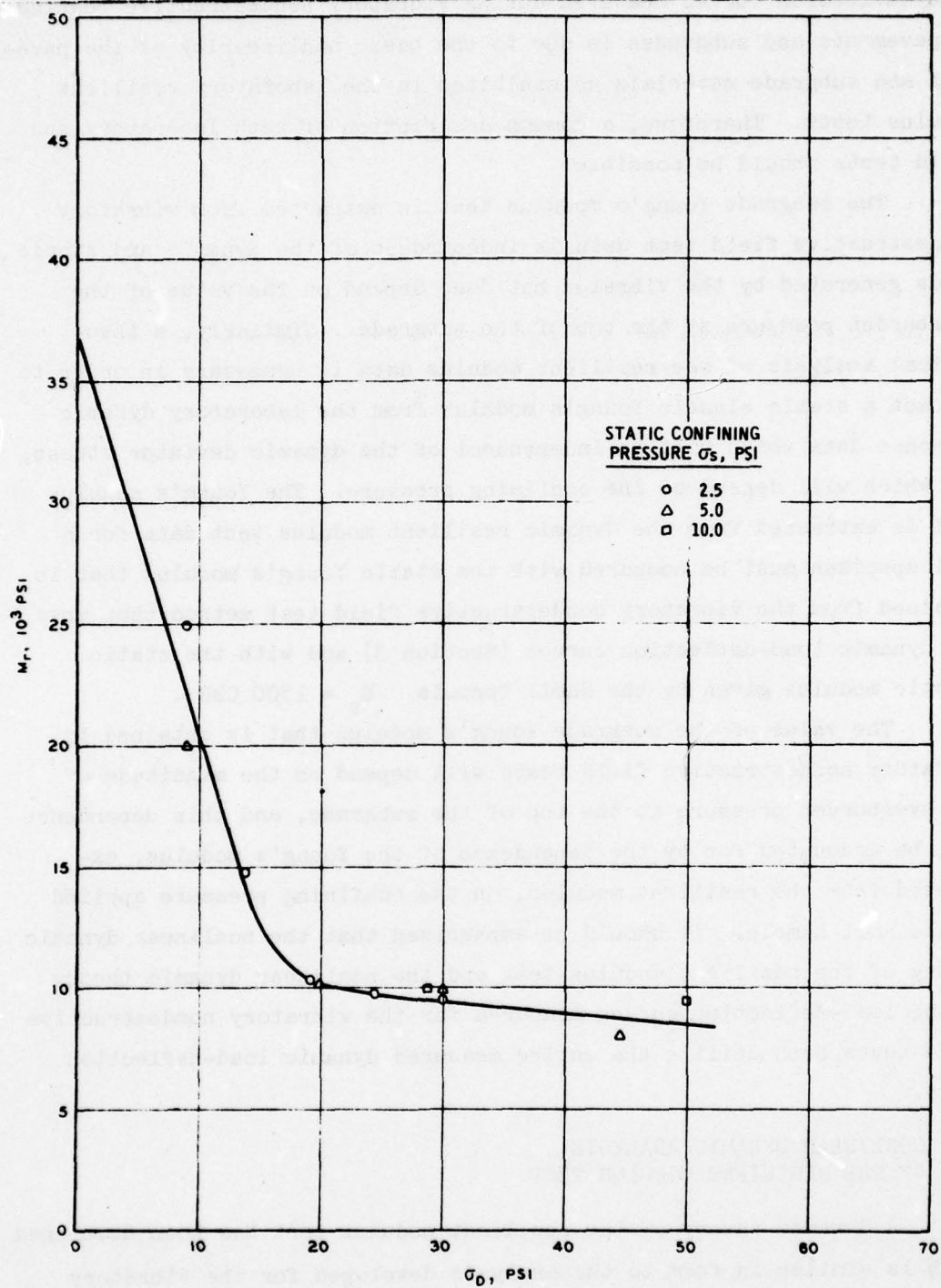


Figure 27. Resilient modulus test results on Alum Creek No. 3 specimen No. 7; $\gamma_d = 109.5$ pcf, $w = 15.0$ percent

load-deflection curves measured during vibratory nondestructive testing of pavements and subgrades is due to the basic nonlinearity of the pavement and subgrade materials as exhibited in the laboratory resilient modulus tests. Therefore, a common description of both laboratory and field tests should be possible.

The subgrade Young's modulus that is extracted from vibratory nondestructive field test data is independent of the dynamic and static loads generated by the vibrator but does depend on the value of the overburden pressure at the top of the subgrade. Similarly, a theoretical analysis of the resilient modulus data is necessary in order to extract a static elastic Young's modulus from the laboratory dynamic response data which will be independent of the dynamic deviator stress, but which will depend on the confining pressure. The Young's modulus that is extracted from the dynamic resilient modulus test data for a soil specimen must be compared with the static Young's modulus that is obtained from the vibratory nondestructive field test method that uses the dynamic load-deflection curves (Section 3) and with the static elastic modulus given by the Shell formula $E_s = 1500 \text{ CBR}$.

The value of the subgrade Young's modulus that is obtained by vibratory nondestructive field tests will depend on the magnitude of the overburden pressure at the top of the subgrade, and this dependence must be accounted for by the dependence of the Young's modulus, extracted from the resilient modulus, on the confining pressure applied to the soil sample. It should be emphasized that the nonlinear dynamic theory of the resilient modulus test and the nonlinear dynamic theory of the load-deflection curves measured for the vibratory nondestructive field tests both utilize the entire measured dynamic load-deflection curve.

4.2 NONLINEAR DYNAMIC ANALYSIS OF THE RESILIENT MODULUS TEST

A dynamic theory of the resilient modulus test has been developed which is similar in form to the analysis developed for the vibratory nondestructive field tests. The basic result of this theory is that the dynamic displacement of the test specimen can be written as

$$\xi_v = \frac{F_D}{S} = \frac{A_C \sigma_D}{S} \quad (4.2)$$

$$S = \sqrt{(k - m\omega^2)^2 + C^2\omega^2} \quad (4.3)$$

where

ξ_v = resilient dynamic displacement on the cylinder end in the axial direction

F_D = dynamic load on the cylinder end in the axial direction

S = dynamic stiffness of the loaded end of the cylinder

A_C = area of the loaded end of the cylinder

σ_D = dynamic stress on the cylinder end in the axial direction

k = spring constant of the loaded end of the cylinder

m = effective mass of the loaded end of the cylinder

ω = effective angular frequency component of the dynamic load applied to the soil sample

C = damping constant of the loaded end of the cylinder

The nonlinear theory of vibrations that was outlined earlier in this paper for the vibratory nondestructive field tests can also be used to calculate the quantities in Equations 4.2 and 4.3. This nonlinear theory shows that the spring constant is given by

$$k = k_0 + \frac{3}{4} b \theta \xi_v^2 + \frac{5}{8} e \eta \xi_v^4 \quad (4.4)$$

$$k_0 = k_{00} + 3b\epsilon_2 x_{ev}^2 + 5e\epsilon_4 x_{ev}^4 \quad (4.5)$$

where b , e , θ , η , ϵ_2 , and ϵ_4 are parameters which characterize the soil sample, and x_{ev} is the resilient static displacement of the soil sample in the axial direction. The coefficients k_{00} , b , and e could be determined from the resilient static stress-strain curve if such a curve could be measured. The resilient static stress-strain curve of the soil sample is determined by

$$F_S = \sigma_S A_C = k_{00} x_{ev} + b x_{ev}^3 + e x_{ev}^5 \quad (4.6)$$

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where

F_S = total static force applied to the cylinder end

σ_S = static confining pressure

The solution of Equations 4.2 and 4.5 can be written as⁴

$$\xi_v = \frac{F_D}{S_0} (1 + \alpha_1 \psi + \alpha_2 \psi^2) \quad (4.7)$$

where

$$S_0 = \sqrt{(k_0 - m\omega^2)^2 + c^2 \omega^2} \quad (4.8)$$

$$\psi = \frac{F_D^2}{S_0^4} = \frac{A_c^2 \sigma_D^2}{S_0^4} \quad (4.9)$$

$$\alpha_1 = -\frac{3}{4} b \theta (k_0 - m\omega^2) \quad (4.10)$$

$$\alpha_2 = \frac{7}{2} \left(\frac{3}{4}\right)^2 b^2 \theta^2 (k_0 - m\omega^2)^2 - S_0^2 \left[\frac{5}{8} n e (k_0 - m\omega^2) + \frac{1}{2} \left(\frac{3}{4}\right)^2 b^2 \theta^2 \right] \quad (4.11)$$

The dynamic stiffness of the soil sample can be obtained from Equations 4.2 and 4.7 to be

$$S = S_0 (1 + \beta_1 \psi + \beta_2 \psi^2) \quad (4.12)$$

$$\beta_1 = -\alpha_1 \quad (4.13)$$

$$\beta_2 = \alpha_1^2 - \alpha_2 \quad (4.14)$$

The quantities necessary for the calculation of the resilient modulus have now been determined.

4.3 CALCULATION OF THE RESILIENT MODULUS

The resilient modulus is defined as the secant slope of the

unloading portion of the dynamic stress-strain curve of the soil sample and is given by

$$M_r = \frac{\sigma_D}{\epsilon_D} = \frac{L}{A_C} \frac{F_D}{\xi_v} = \frac{L}{A_C} S \quad (4.15)$$

where

ϵ_D = dynamic strain in the axial direction

L = length of the soil sample

A_C = area of the end of the cylindrical sample

In Equation 4.15, ξ_v is assumed to describe the unloading portion of the resilient dynamic load-deflection curve of the soil sample. Combining Equations 4.12 and 4.15 gives

$$M_r = M_{ro} (1 + \beta_1 \psi + \beta_2 \psi^2) \quad (4.16)$$

where

$$M_{ro} = \frac{L}{A_C} S_0 \quad (4.17)$$

The quantity M_{ro} is the value of the resilient modulus for a zero value of the dynamic deviator stress; it is, in fact, the intercept points shown in Figures 24-26 for $\sigma_D = 0$. The values of M_{ro} depend on the static confining pressure.

For the low frequency and small mass with which the resilient modulus tests are conducted, the inertial and damping terms in Equations 4.3 and 4.8 can be neglected and the following approximations can be made

$$S \sim k \quad (4.18)$$

$$S_0 \sim k_0 \quad (4.19)$$

The same approximations can be made in Equations 4.10 and 4.11.

Combining Equations 4.5, 4.17, and 4.19 gives the following approximation:

$$M_{ro} \sim e_0 + e_2 x_{ev}^2 + e_4 x_{ev}^4 \quad (4.20)$$

where

$$e_0 = \frac{L}{A_C} k_{00} \quad (4.21)$$

$$e_2 = \frac{L}{A_C} 3b\epsilon_2 \quad (4.22)$$

$$e_4 = \frac{L}{A_C} 5e\epsilon_4 \quad (4.23)$$

The quantities e_0 , e_2 , and e_4 are soil parameters which are independent of the size of the soil sample and machine characteristics. The calculation of the resilient Poisson's ratio requires further study. The quantity M_{ro} gives the intercept values of M_r for $\sigma_D = 0$, while the quantities e_0 , e_2 , and e_4 describe the variation of these intercept values of the resilient modulus with the confining pressure. The term e_0 is the value of the resilient modulus for the case of zero dynamic deviator stress and zero confining pressure; i.e., $\sigma_D = 0$ and $\sigma_S = 0$.

It is also possible to define a tangent slope resilient modulus as follows

$$M_{rt} = \frac{d\sigma_D}{d\epsilon_D} = \frac{L}{A_C} \left(\frac{d\epsilon_v}{dF_D} \right)^{-1} \quad (4.24)$$

where M_{rt} is the tangent resilient modulus. Combining Equations 4.7 and 4.24 gives

$$M_{rt} = M_{ro} (1 + \delta_1 \psi + \delta_2 \psi^2) \quad (4.25)$$

where

$$\delta_1 = -3\alpha_1 \quad (4.26)$$

$$\delta_2 = 9\alpha_1^2 - 5\alpha_2 \quad (4.27)$$

The tangent resilient modulus is analogous to the DSM value measured by vibratory nondestructive tests in the field.

4.4 CALCULATION OF THE STATIC ELASTIC YOUNG'S MODULUS

Equations 4.7, 4.16, and 4.17 show that the parameter M_{ro} describes the linear elastic response of a soil to a dynamic load. The value of M_{ro} depends on the value of the confining pressure at which the resilient modulus test is performed. Both M_r and M_{ro} are dynamic quantities which describe the resilient response of the soil sample to a dynamic load. The Young's modulus, which describes the elastic response of the soil to a static load, must be extracted from the measured values of the dynamic resilient modulus. The Young's modulus of a soil depends on the confining pressure applied to the soil, and its value for a given confining pressure is obtained from Equation 4.6 to be

$$E_s = \epsilon_0 + \epsilon_2 x_{ev}^2 + \epsilon_4 x_{ev}^4 \quad (4.28)$$

where

$$\epsilon_0 = \frac{Lk_{00}}{A_c} \quad (4.29)$$

$$\epsilon_2 = \frac{Lb}{A_c} \quad (4.30)$$

$$\epsilon_4 = \frac{Le}{A_c} \quad (4.31)$$

The parameters ϵ_0 , ϵ_2 , and ϵ_4 depend on the composition and structure of the soil specimen; they are soil parameters. Equation 4.28 gives a general expression for the Young's modulus as a function of confining pressure because x_{ev} is related to σ_s by Equation 4.6. The parameter ϵ_0 is the Young's modulus for an unconfined sample of soil

($\sigma_s = 0$). It is required to determine the coefficients ϵ_0 , ϵ_2 , and ϵ_4 in terms of the measured dynamic resilient modulus data, i.e., it is required to determine E_s from M_r .

The expression for M_r given by Equations 4.16-4.19 characterizes the resilient modulus in terms of σ_D , σ_s , and ω . The parameters required to describe the resilient modulus are k_{00} , b , e , C , θ , η , ϵ_2 , ϵ_4 , m , L , and A_C . These parameters will depend on the type of testing machine, size of soil sample, and the type of soil constituting the soil sample, and therefore the parameters will have to be determined for each type of testing machine. The parameters m , L , and A_C are known immediately from the size and density of the soil specimen. The parameters M_{r0} , β_1 , and β_2 that occur in the expression for M_r given by Equation 4.16 were obtained by fitting Equation 4.16 directly to the measured resilient modulus curves. The parameters k_{00} , b , e , C , θ , η , ϵ_2 , and ϵ_4 were then obtained from M_{r0} , β_1 and β_2 by a trial and error method using Equations 4.8-4.19. The static elastic and the dynamic elastic displacements of the soil sample must also be known. The parameters α_1 , α_2 , δ_1 , and δ_2 are obtained from β_1 and β_2 by using Equations 4.13, 4.14, 4.26, and 4.27. Finally, the parameters e_0 , e_2 , and e_4 are calculated using Equations 4.20-4.23, and the parameters ϵ_0 , ϵ_2 , and ϵ_4 are calculated using Equations 4.29-4.31.

Only a preliminary analysis of the resilient modulus data was performed. The values of the parameters that occur in a resilient modulus test that is described by Equations 4.2-4.23 are given for the case of a specimen of loess in Table 5. The parameters in Table 5 refer to the resilient modulus curves that are presented in Figure 24. In general, the values of the resilient modulus parameters will depend on soil properties such as water content; content of clay, silt, and organic matter; dry density; Atterberg limits; grain size; etc. The brief analysis done here is sufficient to determine only order of magnitude values for these soil parameters. It would be of value to determine the soil parameters that describe the dependence of the resilient modulus on σ_s and σ_D in terms of soil type and composition.

Table 5

Parameters Describing the Dynamic Characteristics of the
Resilient Modulus Laboratory Test on a Sample of Loess
(Figure 24)

Parameter	Unit	Value	Parameter	Unit	Value
A_C	in. ²	6.16	α_1	lb ² /in. ⁴	2.83×10^{13}
L	in.	6.0	α_2	lb ⁴ /in. ⁸	6.07×10^{26}
W	lb	5.0	β_1	lb ² /in. ⁴	-2.83×10^{13}
m	lb sec ² /in.	0.013	β_2	lb ⁴ /in. ⁸	1.94×10^{26}
ω	sec ⁻¹	31.4	θ	Dimensionless	30.0
$m\omega^2$	lb/in.	12.8	n	Dimensionless	50.0
C	lb sec/in.	30.0	ϵ_2	Dimensionless	31.0
C ω	lb/in.	180.0	ϵ_4	Dimensionless	54.0
k_{00}	lb/in.	2.7×10^4	e_0	lb/in. ²	2.6×10^4
k_0	lb/in.	4.0×10^4	e_2	lb/in. ⁴	-1.8×10^9
k	lb/in.	4.0×10^4	e_4	lb/in. ⁶	2.6×10^{15}
b	lb/in. ³	-2.0×10^7	δ_0	lb/in. ²	2.6×10^4
e	lb/in. ⁵	1.0×10^{13}	δ_2	lb/in. ⁴	-1.9×10^7
δ_1	lb ² /in. ⁴	-8.49×10^{13}	δ_4	lb/in. ⁶	1.9×10^{13}
δ_2	lb ⁴ /in. ⁸	4.17×10^{27}			

The value of E_s that is determined by the nonlinear dynamic load-deflection curve method (Section 3) and by the Shell formula $E_s = 1500 \text{ CBR}$ (wave propagation method) includes the effect of the static overburden pressure. Therefore, the values of E_s determined by these two field methods must be compared to the values of E_s that are determined from resilient modulus test data by using Equation 4.28 for a static confining pressure which is equal to the overburden pressure at the top of the subgrade of the pavement locations where the vibratory nondestructive field tests are conducted.

A comparison of M_{ro} given by Equation 4.20 with E_s given by Equation 4.28 shows that these two quantities are not equal. This is reasonable because the former quantity is a dynamic elastic modulus while the latter is a static elastic modulus. However, a comparison of M_{ro} and E_s does show that $\epsilon_0 = e_0$. This is an important conclusion because it shows that the Young's modulus for unconfined soils is equal to the value of the resilient modulus for the condition $\sigma_D = 0$ and $\sigma_S = 0$; i.e., $\epsilon_0 = M_r(\sigma_D = 0, \sigma_S = 0)$. Therefore, ϵ_0 can be obtained directly from resilient modulus test data by extrapolation of the values of M_r to the condition $\sigma_D = \sigma_S = 0$. In other words, the static elastic modulus ϵ_0 can be determined very simply from dynamic resilient modulus test data. It is also true that for a small confining pressure, $M_{ro} \sim E_s$.

Most pavements have a wearing surface, base, and subbase whose total thickness is rarely more than 4 ft. In fact the thickness of a thin pavement is about 1.2 ft, a medium thickness pavement about 2.0 ft, and a thick pavement about 3.5 ft. This means that the overburden pressure at the top of the subgrade is relatively small, being generally less than 4 psi. Therefore, the subgrades of most pavements will have a Young's modulus whose value is given approximately by $E_s \sim \epsilon_0$. Therefore, in general, the Young's modulus E_s of a soil specimen taken from a subgrade can be approximately obtained from an extrapolation of the resilient modulus test data to the condition $\sigma_D = 0$ and $\sigma_S = 0$, because $M_r(\sigma_D = 0, \sigma_S = 0) = \epsilon_0$. For exceptionally thick pavements, the overburden pressure will considerably affect the value of the

subgrade Young's modulus and Equation 4.28 must be used in conjunction with the resilient modulus test data in order to determine E_s .

It has been shown that except for the case of very thick pavements, the Young's modulus of the subgrade can be approximately determined from resilient modulus test data without having to fit Equations 4.2-4.31 of the nonlinear dynamic theory to the measured resilient modulus test data. All that is necessary is sufficient resilient modulus test data to obtain the value of M_r extrapolated to the case $\sigma_D = 0$ and $\sigma_S = 0$, because this immediately gives ϵ_0 which is a good approximation to the value of E_s except at large confining pressures. For the case where the overburden pressure is large enough to make E_s considerably different from ϵ_0 , a full analysis must be done to extract the values of ϵ_0 , ϵ_2 , and ϵ_4 from the resilient modulus test data for use in Equation 4.28.

Undisturbed soil samples were taken from the Alum Creek site and from the Weapons Effects Laboratory (WEL) and Poorhouse sites at WES, and resilient modulus laboratory tests were performed on these samples. The results of these tests appear in Figures 25-27. A rough extrapolation of these curves to the condition $\sigma_D = 0$ and $\sigma_S = 0$ gives an approximate value of the Young's modulus E_s . These extrapolated values agree reasonably with the values of E_s shown in Table 4 that were predicted by the nonlinear dynamic theory applied to the dynamic load-deflection curves that were obtained by vibratory nondestructive testing at these sites. More field and laboratory tests are required to establish a correlation between the values of E_s predicted from the field and laboratory data. Also, resilient modulus tests of much better quality and extending over a greater range of dynamic deviator stress (in the low and high values of σ_D) will be required for accurate laboratory determinations of E_s .

4.5 GENERAL BEHAVIOR OF THE RESILIENT MODULUS $M_r(\sigma_D, \sigma_S)$

The preceding analysis shows that the characteristic shape of the nonlinear dynamic load-deflection curves measured in the field by

the WES 16-kip vibrator is due in part to the basic nonlinear response of the material in the subgrade to dynamic loads. The signs of the coefficients describing the resilient modulus test ($\alpha_1 > 0$, $\alpha_2 > 0$, $\delta_1 < 0$, $\delta_2 > 0$, $\beta_1 < 0$, $\beta_2 > 0$, $b < 0$, and $e > 0$) determine to a large extent the signs of the corresponding coefficients determined from the vibratory nondestructive tests conducted on pavements and subgrades. However, inertial, damping, and frequency effects will affect the values of α_1 and α_2 that are determined by vibratory nondestructive field testing. For the vibratory nondestructive tests done on pavements and subgrades at 15 Hz, it is generally found that $\alpha_1 > 0$ and $\alpha_2 > 0$, which is in agreement with the signs of the corresponding coefficients describing the resilient modulus laboratory test. For frequencies different from 15 Hz and for exceptional pavement cases, it is found that $\alpha_1 > 0$ and $\alpha_2 < 0$ or $\alpha_1 < 0$ and $\alpha_2 > 0$. Therefore, the combination of the large effective mass associated with a pavement and subgrade and the relatively high frequency of operation of the WES 16-kip vibrator can produce a dynamic load-deflection curve which has a shape which is considerably different from the shape of the dynamic load-deflection curve measured in the laboratory during a resilient modulus test.

Because of the finite size of the soil sample for the resilient modulus test, the effective mass of the soil sample is for the purpose of a good approximation equal to the actual mass of the sample. The effective mass that enters the dynamic calculations for the vibratory nondestructive field tests is generally quite large compared to the moving mass of the vibrator because of the large inertial effects associated with the pavement and subgrade. The large effective mass and high frequency of the vibratory nondestructive field tests indicate that the inertial and damping terms are comparable or larger than the elastic effects $m\omega^2 \sim k$ and $C\omega \sim k$. The relatively small mass of the soil sample used for the laboratory resilient modulus tests and the low frequency at which these tests are conducted suggest that for this case $m\omega^2 \ll k$ and $C\omega \ll k$, and the linear and nonlinear elastic properties are measured directly in this test.

The resilient modulus tests combined with the nonlinear dynamic theory of these tests indicate that the static nonlinear elastic coefficients b and e have the signs $b < 0$ and $e > 0$. It is this basic property of soils that is responsible for making the corresponding coefficients determined from field tests exhibit the same signs. The non-zero values of b and e as determined from the resilient modulus test are related to the finite depth of influence of the static stress-strain field in the subgrade beneath a static load placed on the pavement surface. The intrinsic nonlinearity exhibited by the soil during the resilient modulus tests is responsible for the finite depth of influence of the static stress-strain field in an actual soil formation.

The dependence of M_r on the dynamic deviator stress is given theoretically by Equation 4.16 which shows that for small values of σ_D (and $\beta_1 < 0$) the resilient modulus is expected to decrease with increasing values of σ_D , while for larger values of σ_D the resilient modulus is expected either to level off or attain a minimum value before starting to increase with a further increase of the dynamic deviator stress σ_D . Figure 24 indicates that the minimum point may occur, while some experimental resilient modulus test data exist that definitely exhibit this general behavior.^{13,14} The values of M_r may eventually increase with σ_D because of the dynamic compaction of the soil. The decrease of M_r for small values of σ_D may reflect a loosening (or dilatation) of the soil. If this is the case, the coefficients b and ϵ_2 measure the loosening effect while the coefficients e and ϵ_4 measure the soil compaction. In other words, the state of compaction of a soil will determine its degree of nonlinearity under static and dynamic loadings. The coefficients b , e , ϵ_2 , and ϵ_4 are probably important to the description of the liquefaction process in soils because liquefaction is essentially a nonlinear process in soils. These parameters also describe the dilatation and compaction of soils under dynamic loadings. Decreasing values of M_r are not expected for soils with $\beta_1 > 0$.

The experimental resilient modulus data shown in Figures 25-27 show only values of M_r decreasing with an increase of the dynamic

deviator stress σ_D . This is due to the limited range of the dynamic deviator stress that is applied to the soil samples; the dynamic deviator stress was not high enough to observe the increasing values of the resilient modulus. In some cases such as silty sands, only increasing resilient modulus values are observed as a function of dynamic deviator stress.¹⁵ This may be due to the limited range of values of σ_D that were used, because the dynamic deviator stress was not carried low enough into the region where the resilient modulus is a decreasing function of σ_D . On the other hand, it may be the case that silty sands do not exhibit a minimum value of M_r , and M_r may be a monotonic increasing function of σ_D with $\beta_1 > 0$ for this case. Only further experimental resilient modulus test data for silty sands will clarify the situation. The complete description of the nonlinear behavior of a soil specimen requires that a full range of values of the dynamic deviator stress be applied to a soil sample.

For each type of soil, the minimum value of M_r will occur at a specific value of the dynamic deviator stress σ_D which is characteristic of the soil type, density, water content, composition, etc. The value of the dynamic deviator stress at which M_r attains a minimum can be obtained from Equation 4.16 by the condition $dM_r/d\sigma_D = 0$ with the result that

$$\sigma_{DM} = \frac{S_0^2}{A_C} \sqrt{\frac{-\beta_1}{2\beta_2}} \quad (4.32)$$

where σ_{DM} is the value of σ_D for which M_r is a minimum. The value of β_1 must be negative for M_r to exhibit a minimum value. If the situation $\beta_1 > 0$ were encountered, there would be no minimum value of M_r (as may be the case for silty sands) and Equation 4.32 would not be valid in this case.

The value of σ_{DM} will depend on the type of soil, confining pressure, water content, density, frequency of applied dynamic load at which the resilient modulus test is conducted, etc. Figure 24 shows that for the sample of loess at a confining pressure of $\sigma_S = 40$ psi,

the value of the dynamic deviator stress at which M_r attains a minimum value is $\sigma_{DM} = 60$ psi. Because S_0 is an increasing function of the confining pressure, Equation 4.32 shows that σ_{DM} is an increasing function of the confining pressure as indicated roughly in Figure 24. The value of σ_{DM} is a good measure of the liquefaction potential of a soil subjected to a dynamic stress σ_D . If $\sigma_D < \sigma_{DM}$, the soil will dilate and possibly liquefy while if $\sigma_D > \sigma_{DM}$ the soil will be compacted and probably will not liquefy. The parameter σ_{DM} is a good indicator of the stability of a subgrade soil under the action of known dynamic loads.

When an aircraft operates on a rough pavement, dynamic forces are produced which in turn create dynamic stresses in the subgrade. Depending on the relative values of σ_D and σ_{DM} , the subgrade soil can either dilate or contract under dynamic loads, thereby producing an eventual failure of the subgrade. The laboratory measured value of σ_{DM} can be used to predict the future behavior of a subgrade soil under the action of a known dynamic loading. Therefore, although the basic use of the measured resilient modulus of a soil is the determination of the Young's modulus $E_s(\sigma_s)$, an important secondary use of M_r would be the determination of the density variation of subgrade soils under dynamic loading.

There is also a minimum value of the resilient modulus when it is considered as a function of the static confining pressure. For the case of $\sigma_D = 0$, the minimum value can be calculated from Equation 4.2 by calculating $dM_{ro}/d\sigma_s = 0$ with the result that

$$x_{em} = \sqrt{\frac{-3b\epsilon_2}{10e\epsilon_4}} \quad (4.33)$$

where x_{em} is the static elastic displacement of the soil sample at the static confining pressure where M_{ro} is a minimum. The value of b is negative. The value of this characteristic confining pressure is then obtained by placing Equation 4.33 into Equation 4.6. The static confining pressure at which the minimum in M_{ro} occurs is so small (~ 1.0 psi) that it is probably of no practical interest.

4.6 RELATIONSHIP OF THE LOAD-CARRYING CAPACITY OF A PAVEMENT AND THE RESILIENT MODULUS OF THE SUBGRADE

The allowable load (bearing capacity, load-carrying capacity) refers to the maximum static load that can be applied to a pavement for a specified number of repetitions before failure occurs.³ Because a static load is involved, the pressure-dependent Young's modulus of the subgrade E_s is entered in the static layered elastic computer programs that theoretically predict the allowable load for a pavement. It is not the resilient modulus of the subgrade M_r that is entered in the static layered elastic computer programs (Chevron Program and Shell Program), because M_r describes the response of a material to a dynamic load. The limiting stress and strain criteria that govern the performance of pavements and subgrades refer to static stress and strain levels.¹⁵ These limiting values of static stress and strain must be used in conjunction with Young's modulus values to determine the allowable static load for a pavement.

If the allowable static load of a pavement is required, the values of $M_r(\sigma_D, \sigma_S)$ must be used to determine the Young's modulus of the subgrade $E_s(\sigma_S)$ by the method outlined in this section. The function $E_s(\sigma_S)$ is then used in the static layered elastic computer programs to calculate the static load-carrying capacity of a pavement.

5. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

5.1 SUMMARY

Of much importance to pavement engineers is an estimation of the strength and condition of the subgrade of a pavement without having to drill holes through the pavement to have direct access to the subgrade. The layered elastic approach to the problem of calculating the load-carrying capacity and the overlay thickness of a pavement requires the value of the subgrade Young's modulus in addition to the elastic moduli of the pavement layers. The nonlinear dynamic response model that is developed to describe the dynamic load-deflection curves that are measured at a pavement surface can be used to eliminate the extraneous effects of the static and dynamic loads produced by the WES 16-kip vibrator and to determine quickly and accurately the value of the subgrade Young's modulus.

The nonlinear dynamic pavement response model that is presented in this report gives a quantitative description of the dynamic response of a pavement surface under the action of the dynamic and static load applied to the pavement surface by the WES 16-kip vibrator. The model parameters (spring constants, effective mass, damping constant, and finite depth of influence of the static load) have been determined as a function of pavement strength as represented by the measured DSM. The nonlinear pavement response model gives a theoretical expression for the pavement response in terms of these parameters and in terms of the elastic moduli of the pavement and subgrade. For a suitable choice of the elastic moduli of the pavement layers, it is possible to predict the value of the subgrade Young's modulus from the dynamic load-deflection curve measured at the pavement surface.

The nonlinear dynamic model has also been applied to the dynamic resilient modulus test, and the resilient modulus has been analytically characterized as a function of static confining pressure, dynamic deviator stress, and coefficients that describe the material properties of the soil. The Young's modulus of the subgrade soil sample can be extracted from the dynamic resilient modulus data, and this Young's

modulus can be compared with the subgrade Young's modulus that is predicted from vibratory nondestructive field tests.

5.2 CONCLUSIONS

5.2.1 FREQUENCY RESPONSE METHOD

The study of the use of the frequency response curves for determining the elastic modulus of the subgrade gave the following conclusions:

- a. A relatively simple linear spring model can be developed which can describe a frequency response curve having one dominant peak; however, this linear model is inadequate to relate the frequency response curve to the Young's moduli of the subgrade and pavement (Sections 2.3 and 2.4.1).
- b. The three parameters of the linear spring model k , m , and C can be determined in terms of the position and size of the deflection peak. For most pavements the peak frequency is about 15-20 Hz and the resonance frequency is about 15-20 Hz (Sections 2.3 and 3.3.8).
- c. The effective mass is an increasing function of the pavement strength and is generally much larger than the above-surface moving mass of the vibrator that is used to excite the pavement surface. The spring constant and damping constant are also increasing functions of the pavement strength (Section 2.4.1).
- d. The value of the spring constant k obtained from the dynamic data is generally much larger than that predicted by the Chevron linear layered elastic program for a pavement system whose subgrade Young's modulus is given by $E_s = 1500 \text{ CBR}$. This is due to the nonlinear nature of the pavement response to dynamic load which implies that the spring constant k depends on the magnitudes of the static and dynamic loads exerted by the vibrator on the pavement surface (Section 2.4.1).
- e. If the value of the spring constant k that is determined from the frequency response spectrum data is used in conjunction with the linear layered elastic Chevron Program to determine the value of the subgrade Young's modulus, the predicted value of the subgrade Young's modulus is several times larger than the value given by the $E_s = 1500 \text{ CBR}$ rule. This is due to the extraneous effects of the static and dynamic loads on the value of the spring constant k . The spring constant k is not directly related to the Young's moduli of the subgrade and pavement layers (Section 2.4.1).
- f. The Young's modulus of the subgrade cannot directly be obtained from the linear model of the frequency response

spectrum, and a nonlinear dynamic model seems to be required to determine the subgrade Young's modulus (Sections 2.4.1 and 2.5).

5.2.2 DYNAMIC LOAD-DEFLECTION CURVE METHOD

The nonlinear dynamic pavement response model that is used to describe the dynamic load-deflection curves that are measured on pavements and subgrades using the WES 16-kip vibrator yields the following conclusions:

- a. A single-mass nonlinear spring model can be developed which adequately describes the measured dynamic load-deflection curves and predicts the value of the subgrade Young's modulus (Section 3.2).
- b. Thirteen parameters are required for the nonlinear spring model: k_{00} , b , e , θ , η , ϵ_2 , ϵ_4 , l_0 , l_2 , l_4 , κ , m , and C . These parameters have been determined as a function of pavement strength as represented by the measured DSM value (Section 3.3 and Figures 6-20).
- c. Damping and inertial terms make significant contributions to the dynamic response of a pavement, and the damping and inertial terms vary greatly with the type of pavement. Both C and m are larger for stiffer pavements, but the damping ratio D decreases for increasing values of the measured DSM (Section 3.3.3 and Figures 8-10).
- d. The spring constant k determined from dynamic data depends on the dynamic and static loads generated by the vibrator and cannot be identified with the static elastic spring constant that would be obtained from static plate bearing tests or from layered linear elastic theory computer programs (Sections 3.2.3 and 3.3.1, and Figure 6).
- e. The static linear elastic spring constant k_{00} and the static nonlinear elastic spring constant k_0 are increasing functions of the measured DSM, and, therefore, the measured DSM of a pavement can be used as a measure of the static stiffness of a pavement or subgrade. The spring constant k_0 depends on the static load applied by the vibrator to the pavement surface (Sections 3.3.1 and 3.3.4, and Figures 7 and 11).
- f. The static elastic load-deflection curves are determined by parameters k_{00} , b , and e . The nonlinear parameters b and e make a significant contribution to the shape of these curves and to the value of the elastic part of the reaction modulus determined from plate bearing tests on subgrades and pavements. The parameters k_{00} , b , and e are generally

increasing functions of the measured DSM. The predicted static elastic displacement of the pavement under the vibrator baseplate is a decreasing function of the measured DSM (Section 3.3.4 and Figures 11-14).

- g. The nonlinearity of the static and dynamic load-deflection curves can be related to the finite depth of influence of the static stress and strain field in the subgrade. The parameters l_0 , l_2 , and l_4 , which describe the finite depth of influence of the static stress-strain field, are found to be increasing functions of the measured DSM (Sections 3.2.4 and 3.3.5, and Figures 15-17).
- h. The lateral spreading of the stress and strain distribution in the pavement and subgrade is described by the parameter κ and tends to increase for increasing values of the measured DSM (Sections 3.2.4 and 3.3.6, Figure 18, and Reference 4).
- i. There are significant interaction terms involving the dynamic and static displacement in the equations of motion of the pavement surface beneath the vibrator baseplate, and four parameters θ , η , ϵ_2 , and ϵ_4 are required to quantify these interaction terms (Section 3.2.1 and 3.3.7, and Figures 19 and 20).
- j. The nonlinear dynamic theory of pavement response shows that there is a critical frequency f_c for which the measured nonlinear dynamic load-deflection curves tend to be less curved than the load-deflection curves measured at other frequencies. The critical frequency decreases with increasing values of the measured DSM but remains in the neighborhood of 15 hz for medium-strength and strong pavements (Sections 3.2.2 and 3.3.8, and Figure 21).
- k. The subgrade Young's modulus can be determined directly from the measured dynamic load-deflection curve, and a reasonable agreement with the rule $E_s = 1500 \text{ CBR}$ is obtained. The subgrade modulus depends on the static overburden pressure produced by the layers of pavement above the subgrade (Section 3.4).

5.2.3 LABORATORY CONFIRMATION OF THE SUBGRADE ELASTIC MODULUS VALUE

The following conclusions can be obtained from a consideration of a nonlinear dynamic theory of the laboratory resilient modulus test:

- a. A nonlinear dynamic theory of the resilient modulus laboratory test can be developed which is analogous to the nonlinear theory of the vibratory nondestructive tests conducted on pavements and subgrades (Section 4.2).

- b. The resilient modulus is expressed analytically as a function of confining pressure, dynamic deviator stress, and the frequency of application of the dynamic loading in the axial direction. The resilient modulus is a dynamic modulus which describes the response of a material to a dynamic load (Section 4.3).
- c. It is possible to extract the static elastic Young's modulus from the measured resilient modulus test data. It is the Young's modulus that enters the layered elastic computer programs that calculate the allowable load-carrying capacity of a pavement. It is this Young's modulus that must be compared with the subgrade Young's modulus that is predicted from vibratory nondestructive field tests and with the subgrade Young's modulus given by the Shell wave propagation relationship $E_s = 1500 \text{ CBR}$ (Section 4.4).
- d. The Young's modulus that is extracted from the resilient modulus test is a function of the confining pressure. The Young's modulus under unconfined conditions can be easily obtained from M_r by extrapolating M_r to the condition $\sigma_D = 0$ and $\sigma_S = 0$ (Section 4.4).
- e. The resilient modulus may have a minimum value at a specific value of the dynamic deviator stress which depends on the soil type and structure (Section 4.5).
- f. The allowable static load of a pavement can be determined by the layered elastic theory if the subgrade modulus that is used is the static elastic Young's modulus that is extracted from the resilient modulus test (Section 4.6).

5.3 RECOMMENDATIONS

The use of layered elastic theory computer programs to predict the allowable load-carrying capacity of a pavement requires an accurate value of the subgrade Young's modulus and accurate values of the elastic moduli of all the pavement layers that occur on top of the subgrade. This report develops the capability of predicting the Young's modulus of the subgrade using vibratory nondestructive testing techniques, and presents a method of laboratory confirmation of the field tests results through measurement and analysis of laboratory resilient modulus data. Further experimental and theoretical work is necessary to apply the basic techniques and conclusions of this report to the problem of calculating the allowable load-carrying capacity of a pavement.

5.3.1 DETERMINATION OF SUBSURFACE STRUCTURE

The determination of the subgrade Young's modulus by the vibratory nondestructive testing technique requires a knowledge of the elastic moduli of the pavement layers above the subgrade. The determination of the allowable load-carrying capacity of a pavement by the method of layered elastic theory requires the elastic moduli of all pavement layers as well as the Young's modulus of the subgrade. Therefore, the Young's moduli of the pavement layers are used twice in the procedure for calculating the allowable load-carrying capacity of a pavement. In view of this it is recommended that:

- a. Vibratory nondestructive tests be developed which will accurately determine the values of the Young's moduli of all pavement layers. This will include wave propagation methods such as the Rayleigh wave dispersion technique.
- b. The development of a reliable method of estimating the Young's modulus of the material in each pavement layer in terms of its composition and structure be undertaken.

5.3.2 LABORATORY CONFIRMATION OF FIELD TEST DATA

A complete connection between the resilient modulus laboratory tests and the vibratory nondestructive field tests has not yet been accomplished. But the results of a preliminary theoretical study show that it is possible to apply a nonlinear dynamic theory to the resilient modulus laboratory test to determine the static elastic Young's modulus of a subgrade soil, and to compare this value with the Young's modulus value predicted by the nonlinear dynamic analysis of the vibratory nondestructive field test data and with the Young's modulus predicted by the Shell formula $E_s = 1500 \text{ CBR}$.

A nonlinear dynamic analysis of laboratory resilient modulus test data requires a complete determination of the resilient modulus function $M_r(\sigma_D, \sigma_s)$ over an extended range of values of σ_D and σ_s . Several soil parameters are derived from a nonlinear dynamic analysis of the resilient modulus test data, including the Young's modulus parameters ϵ_0 , ϵ_2 , and ϵ_4 (Section 4.4). It is thought

that further resilient modulus tests performed on pavement and subgrade materials will be of value to allowable load calculations. An analysis of these test will include:

- a. The complete determination of the function $M_r(\sigma_D, \sigma_S)$ for a full range of values of σ_D and σ_S sufficient to observe the minimum value of M_r that may occur at a specific value of σ_D , and sufficient to allow an accurate extrapolation of M_r to the condition $\sigma_D = 0$ and $\sigma_S = 0$.
- b. The application of the nonlinear dynamic response theory to the resilient modulus test in order to determine the soil parameters of this model in terms of soil characteristics such as water content, dry density, soil composition, etc. Specifically, the determination of the Young's modulus coefficients E_0 , E_2 , and E_4 is important in order to determine the dependence of Young's modulus on the static confining pressure for any soil type.
- c. It is suggested that these parameters be obtained from resilient modulus tests on many undisturbed soil specimens taken from the subgrades of pavements where vibratory nondestructive tests have been conducted. In this way, a connection between laboratory and field test data can be made.
- d. It should be possible to extrapolate the Young's modulus function to values of σ_S corresponding to the static confining pressure produced in the subgrade by the static weight of an aircraft.
- e. The resilient modulus may be used to determine the stability of subgrade soils under the action of dynamic loads. It suggested that the importance of the parameter σ_{DM} be examined for this purpose.

APPENDIX A: DOCUMENTATION OF THE WES
DYNAMIC FREQUENCY RESPONSE PROGRAM

Program Identification

1. Program Title: WES Dynamic Frequency Response Program
2. Program Code Name: ECNST
3. Writer: Richard A. Weiss
4. Organization: U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi 39180
5. Date: March 1976
6. Source Language: FORTRAN IV
7. Availability: Complete program listing is available at WES.
8. Abstract: Program calculates the linear spring model parameters: spring constant (k), effective mass (m), and damping constant (C) from a measured frequency response spectrum.

Engineering Documentation

9. Narrative Description: Program ECNST, "WES Dynamic Frequency Response Program," calculates the linear spring model parameters k, m, and C from the deflection peak of a measured frequency response spectrum. The required measured quantities are the peak frequency, peak amplitude, and the ratio of the peak amplitude to the amplitude at a frequency in the neighborhood of the peak frequency.

10. Method of Solution: A linear harmonic oscillator model was developed to describe the measured frequency response spectrum of the pavement surface. The governing equation of motion of the pavement surface is assumed to be

$$m\ddot{A} + C\dot{A} + kA = F_D$$

where

- m = effective mass of pavement
- A = dynamic deflection of the pavement surface
- C = damping constant
- k = spring constant
- F_D = dynamic load applied to pavement surface

The computer program calculates the parameters k , m , and C by solving the following equations:

$$k = 4\pi^2 f_M^2 m \left[1 + \left(\frac{F_D}{4\pi^2 m f_M^2 A_M} \right)^2 \right]^{1/2}$$

$$D^2 = \frac{1}{2} - \frac{1}{2} \left[1 + \left(\frac{F_D}{4\pi^2 m f_M^2 A_M} \right)^2 \right]^{1/2}$$

$$\frac{A_m}{A} = \frac{\left[\left(1 - \frac{m\omega^2}{k} \right)^2 + \frac{4mD^2\omega^2}{k} \right]^{1/2}}{\left[\left(1 - \frac{m\omega_M^2}{k} \right)^2 + \frac{4mD^2\omega_M^2}{k} \right]^{1/2}}$$

The required measured quantities are f_M , A_M , and A_M/A . The simultaneous solution of these three equations gives k , m , and D from which C is obtained by $C = 2D\sqrt{km}$.

11. Program Capabilities: The model is based on a single-mass linear harmonic oscillator and can only describe a frequency response spectrum that has one dominant deflection peak.

12. Data Inputs: The program requires A_M , f_M , and A_M/A .

13. Printed Output: The printed output consists of the three model elements k , m , and C .

14. Computer Equipment: Program ECNST was developed on a GE-400 computer.

15. Source Program: The source listings for program ECNST are available at WES.

APPENDIX B: DOCUMENTATION OF THE WES NONLINEAR
DYNAMIC LOAD-DEFLECTION PROGRAM

Program Identification

1. Program Title: WES Nonlinear Dynamic Load-Deflection Programs
2. Program Code Name: NLIN, SUBE
3. Writer: Richard A. Weiss
4. Organization: U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi 39180
5. Date: March 1976
6. Source Language: FORTRAN IV
7. Availability: Complete program listing is available at WES.
8. Abstract: Program NLIN fits a polynomial expression to the measured dynamic load-deflection curves and determines the linear and nonlinear parameters of the dynamic model that describes the nonlinear dynamic load-deflection curves. The program SUBE calculates the value of the subgrade Young's modulus by requiring the nonlinear dynamic model to describe the measured dynamic load-deflection curves and to reproduce the measured DSM value.

Engineering Documentation

9. Narrative Description: Programs NLIN and SUBE, "WES Nonlinear Dynamic Load-Deflection Programs," calculate the basic parameters of the nonlinear dynamic response theory of pavements. The program NLIN is a research program that is used to determine the nonlinear spring model parameters. The program SUBE is used to calculate the value of the subgrade Young's modulus from the measured dynamic load-deflection curves and the elastic moduli of the pavement layers.

10. Method of Solution: A nonlinear oscillator model was developed to describe the dynamic load-deflection curve measured at the pavement surface. The governing equation of motion is assumed to be

$$m\ddot{x} + C\dot{x} + k_{00}x + bx^3 + ex^5 = F_S + F_D$$

where

m = effective mass
x = total elastic displacement, static plus dynamic
C = damping constant
 k_{00} = linear spring constant
b = third-order nonlinear elastic coefficient
e = fifth-order nonlinear coefficient

The equation of motion is solved by a series expansion, and the coefficients k_{00} , b, and e are determined by matching the series expansion to the measured dynamic load-deflection curve.

11. Program Capabilities: The model is based on a single-mass nonlinear oscillator with third- and fifth-order nonlinear terms and can be used to describe and evaluate a nonlinear dynamic load-deflection curve measured on a pavement or subgrade and predict the subgrade Young's modulus from the measured data. The program is valid only for values of DSM in the range $300 < \text{DSM} < 6500$.

12. Data Inputs: The programs require a tabulation of values of dynamic load and dynamic deflection as determined from the measured dynamic load-deflection curve, the elastic moduli of the pavement layers, and the measured DSM value.

13. Printed Output: The printed output of the program NLIN consists of the values of the parameters k_{00} , b, and e, as well as assorted spring constants, dynamic stiffness values, depth of influence of the static stress-strain field, static displacement of the surface, effective mass, and the damping constant. The printed output of the program SUBE are the predicted values of the subgrade Young's modulus.

14. Computer Equipment: Programs NLIN and SUBE were developed on a GE-400 computer.

15. Source Program: The source listings for programs NLIN and SUBE are available at WES.

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LIST OF SYMBOLS

a	Radius of vibrator baseplate
a_1, a_2, a_3, a_4	Coefficients
A	Dynamic amplitude of motion of the pavement surface as determined by the linear elastic model
\dot{A}	Velocity of pavement surface
\ddot{A}	Acceleration of pavement surface
A_C	Area of end face of cylindrical soil sample
A_M	Peak dynamic amplitude of the pavement surface
b	Third order nonlinear pavement and subgrade parameter
C	Damping constant of the vibrator-pavement-subgrade system
CBR	California Bearing Ratio
D	Damping ratio
DSM	Dynamic stiffness modulus
e	Fifth order nonlinear pavement and subgrade parameter
e_0, e_2, e_4	Expansion coefficients for the resilient modulus at zero dynamic deviator stress
$e^{i(\omega t - \Lambda)}$	Complex number notation for a sinusoidal time dependence
E_s	Young's modulus of subgrade
$\epsilon_0, \epsilon_2, \epsilon_4$	Expansion coefficients of the Young's modulus
f_c	Critical frequency
f_M	Peak frequency
f_R	Resonance frequency
$F(t)$	Dynamic load of vibrator
$F_D(\omega)$	Magnitude of the sinusoidal dynamic force applied to the pavement surface
F_S	Static load applied to the pavement surface
g	Acceleration of gravity
G	Shear modulus
$J(\omega)$	Measured value of the ratio of the peak amplitude to the amplitude at a neighboring frequency

$J(k,m,C,\omega) = J(m,\omega)$	Theoretical value of the ratio of the peak dynamic amplitude to the amplitude at a neighboring frequency
k	Dynamic spring constant of a pavement or subgrade
k_0	Effective static spring constant that appears in the dynamic equations of motion
k_T	Theoretical value of the static spring constant as predicted by a layered linear elastic computer program
k_{00}	Linear elastic spring constant for a nonlinear pavement
l	Finite depth of influence of the static strain field
l_0, l_2, l_4	Coefficients of the power series expansion of the finite depth of influence
L	Length of the soil sample
m	Lumped effective mass of pavement and subgrade
m_v	Mass of moving weight of the vibrator
M_r	Resilient modulus
M_{r0}	Resilient modulus value for the case of zero dynamic deviator stress
S	Dynamic stiffness of pavement or subgrade
S_0	Dynamic stiffness for zero dynamic load
t	Time
W_v	Weight of moving mass of vibrator
x	Total elastic deflection of the pavement surface beneath the vibrator baseplate
x_e	Static elastic deflection of the pavement surface
x_{em}	Static elastic displacement of soil sample at the static confining pressure where M_{r0} is a minimum
x_{ev}	Static elastic deflection of soil sample in a resilient modulus test
$\alpha_1, \alpha_2 \dots \alpha_n$	Coefficients appearing in the power series expansion of the amplitude of the dynamic deflection of the pavement surface
$\beta_1, \beta_2 \dots \beta_n$	Coefficients for the power series expansion of the dynamic stiffness

δ	Function of the expansion coefficients of the finite depth of influence
δ_1, δ_2	Expansion coefficients of the resilient modulus test
ϵ_D	Dynamic strain in axial direction of the soil specimen in the resilient modulus test
$\theta, \eta, \epsilon_2, \epsilon_4$	Parameters describing the cross product terms of the static and dynamic deflection terms in the equation of motion of the pavement surface
κ	Ratio of the radius of the lower base to the radius of the upper area of the frustum of the cone of stress and strain
ν	Poisson's ratio
ξ	Dynamic elastic deflection of the pavement surface beneath the vibrator baseplate measured from the static equilibrium deflection
ξ_v	Resilient dynamic displacement of the cylinder end in the vertical direction
σ	Total stress along the axis of the soil sample for the resilient modulus test
σ_D	Dynamic deviator stress in the axial direction of the soil sample
σ_S	Static confining pressure on the soil sample
σ_{DM}	Value of σ_D for which M_r is a minimum
σ_{OB}	Overburden pressure on the soil at the top of the subgrade
ψ	$F_{D0}^2 S^{-4}$
Ψ	Volume factor for the frustum of the cone of stress and strain
ω	Angular frequency
ω_M	Peak angular frequency
ω_R	Angular frequency at resonance
Λ	Phase angle between the dynamic load applied to the pavement surface and the dynamic deflection of the pavement surface